

Computational Times

Data from the algorithms

We present the computational data related to the runs of the algorithmic procedure for the classification of number fields with bounded discriminant. Every presented table depends on:

- Signature of the investigated fields;
- Value of the parameter N , which is the absolute value of either a_8 or a_9 , depending on the degree of the fields being either 8 or 9;
- Parity of the evaluation in 1 of the polynomials $p(x)$ produced in the algorithm: the difference is chosen in order to expose that almost every polynomial providing the suitable fields derives from a case where $p(1)$ is odd.

Every table presents the following values:

- A value for the coefficient a_1 , ranging from 0 to -4 ;
- A value for the coefficient a_8 (a_9) which can be either N or $-N$;
- The time spent for the computations in the run launched with the given data;
- The number of polynomials created in the corresponding run;
- The number of polynomials in the run which survived the test;
- The number of suitable fields (up to isomorphism) detected in the corresponding run.

The algorithm was launched on the computational cluster of the Institut de Mathématiques de Bordeaux, on the cluster HORIZON of Dipartimento di Matematica dell'Università degli Studi di Milano and on the cluster INDACO of Università degli Studi di Milano.

Remark 1. At some point in the tables, the number of polynomials surviving the test decrease drastically respect to the previous data. This difference is due to the introduction of the `coredisc()` condition which was implemented during the Atelier PARI/GP which took place in Bordeaux during January 2019.

Signature (2,3)

Table 1: $N = 1, p(1)$ odd

a_1	a_8	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
0	1	9 m, 36 s	86675321	708698	46
0	-1	12 m, 47 s	87266074	1405030	51
1	1	18 m, 36 s	130082902	1365507	49
1	-1	23 m, 19 s	130837280	2422481	51
2	1	20 m, 40 s	125942700	1553596	53
2	-1	25 m, 15 s	126723535	2652440	53
3	1	28 m, 53 s	126267044	1569951	50
3	-1	32 m, 0 s	126759623	2495219	51
4	1	46 m, 8 s	153516283	1283199	56
4	-1	48 m, 39 s	153758399	1777727	54

Table 2: $N = 1, p(1)$ even

a_1	a_8	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
0	1	3 m, 55 s	21428574	1066	0
0	-1	4 m, 18 s	21563567	2662	1
1	1	8 m, 4 s	33563227	3061	0
1	-1	7m, 53 s	33744575	6067	1
2	1	10 m, 34 s	32889347	4734	0
2	-1	10 m, 9 s m	33079350	8864	0
3	1	16 m, 2 s	33130978	3708	0
3	-1	15 min, 48 s	33272615	5673	1
4	1	34 m, 51 s	35142317	3522	0
4	-1	35 m, 34 s	35190489	6572	1

Signature (4,2)

Table 3: $N = 1, p(1)$ odd

a_1	a_8	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
0	1	1 h, 37 m, 44 s	1277978266	1120766	31
0	-1	1 h, 40 m, 25 s	1280596696	601865	27
1	1	3h, 15 m, 10 s	1959119583	2706272	36
1	-1	3 h, 9 m, 1 s	1962391324	1222348	35
2	1	3h, 17 m, 26 s	1643511207	3045442	33
2	-1	3 h, 11 m, 9 s	1646498913	1047607	34
3	1	4 h, 5 m, 5 s	1618437106	3664299	35
3	-1	4 h, 54 m, 15 s	1621282936	1652741	35
4	1	6 h, 29 m, 48 s	1753331082	3606483	36
4	-1	6 h, 26 m, 44 s	1754659076	2721034	36

Table 4: $N = 1, p(1)$ even

a_1	a_8	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
0	1	1 h, 0 m, 50 s	306988761	2726	0
0	-1	55 m, 56 s	307590353	1310	0
1	1	2 h, 22 m, 34 s	521827017	6448	0
1	-1	2 h, 26 m, 58 s	522749091	2307	0
2	1	2h, 36 m, 34 s	400989306	7460	1
2	-1	2 h, 38 m, 11 s	401699553	2417	0
3	1	3 h, 34 m, 0 s	391723479	9779	0
3	-1	3 h, 8 m, 3 s	392393757	4991	0
4	1	5 h, 54 m, 50 s	455177646	11844	0
4	-1	5 h, 56 m, 39 s	455503227	9286	0

Signature (6,1)

Table 5: $N = 1, p(1)$ odd

a_1	a_8	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
0	1	19 h, 27 m, 3 s	15878290619	5853	4
0	-1	19 h, 40 m, 49 s	15889378178	17834	3
1	1	30 h, 50 m, 36 s	22999940351	9226	7
1	-1	29 h, 36 m, 42 s	23009638906	23145	4
2	1	34 h, 2 m, 24 s	21655304221	12714	6
2	-1	33 h, 11 m, 37 s	21670342460	20288	5
3	1	41 h, 59 m, 23 s	20669410904	18738	7
3	-1	41 h, 13 m, 59 s	20684431832	18290	4
4	1	80 h, 58 m, 12 s	21915140316	22900	5
4	-1	80 h, 52 m, 2 s	21926999910	12289	4

Table 6: $N = 1, p(1)$ even

a_1	a_8	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
0	1	12 h, 5 m, 31 s	4006752728	3	0
0	-1	10 h, 28 m, 57 s	4009593574	66	0
1	1	22 h, 44 m, 36 s	5550034483	5	0
1	-1	19 h, 47 m, 17 s	5552358577	83	0
2	1	25 h, 54 m, 10 s	5324018404	7	0
2	-1	25 h, 34 m, 37 s	5327751601	67	0
3	1	35 h, 44 m, 8 s	5281851819	19	0
3	-1	35 h, 8 m, 58 s	5285738866	58	0
4	1	78 h, 38 m, 51 s	5334734422	27	0
4	-1	77 h, 45 m, 14 s	5337685001	34	0

Signature (1,4)

Table 7: $N = 1, p(1)$ odd, $a_1 = 0, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
12	2 h, 49 m, 27 s	715088104	3089563	11
10	3 h, 9 m, 44 s	1201171516	4797023	8
8	5 h, 10 m, 20 s	1943438943	7976985	11
6	7 h, 4 m, 32 s	2462855901	10430195	18
4	7 h, 29 m, 11 s	2517993623	11044870	22
2	8 h, 8 m, 9 s	2643279263	11852922	24
0	7 h, 57 m, 47 s	2566834761	11891205	22
-2	8 h, 23 m, 43 s	2644627400	12215694	20
-4	7 h, 47 m, 57 s	2478820937	11490331	19
-6	8 h, 35 m, 18 s	2363593813	10583390	9
-8	7 h, 20 m, 52 s	1997283285	8714424	14
-10	5 h, 59 m, 20 s	1675247720	6703701	8
-12	5 h, 8 m, 50 s	1252879071	4710494	6

Table 8: $N = 1, p(1)$ odd, $a_1 = 0, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
12	3 h, 7 m, 18 s	715620502	3270302	9
10	4 h, 48 m, 9 s	1202988900	5170865	9
8	6 h, 21 m, 20 s	1943729657	8660083	19
6	7 h, 39 m, 34 s	2462847356	11644867	24
4	10 h, 20 m, 3 s	2519779091	12386930	29
2	10 h, 37 m, 56 s	2645640318	13281743	20
0	10 h, 8 m, 56 s	2568660876	13093053	36
-2	10 h, 11 m, 7 s	2645581822	13039021	29
-4	9 h, 9 m, 20 s	2478330191	11718153	23
-6	8 h, 19 m, 25 s	2361565243	10270425	25
-8	6 h, 5 m, 50 s	1994470283	8063187	18
-10	5 h, 6 m, 30 s	1672690064	5958283	5
-12	3 h, 36 m, 22 s	1251334957	3856259	11

Table 9: $N = 1, p(1)$ odd, $a_1 = -1, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
13	4 h, 41 m, 12 s	952047563	3866319	4
11	6 h, 37 m, 48 s	1558616154	5433798	19
9	10 h, 23 m, 20 s	2494921045	9435874	19
7	15 h, 22 m, 49 s	3557147123	14678721	30
5	16 h, 37 m, 9 s	3785641496	16679614	38
3	16 h, 46 m, 58 s	3828884005	17164254	41
1	17 h, 0 m, 10 s	3832830065	17940374	36
-1	16 h, 54 m, 12 s	3822877795	17923143	37
-3	18 h, 31 m, 58 s	3762853483	17907633	35
-5	13 h, 15 m, 17 s	3615234404	16778405	30
-7	10 h, 26 m, 50 s	3269688842	14600904	21
-9	11 h, 22 m, 23 s	2769039197	11623983	18
-11	9 h, 34 m, 16 s	2096497567	8258592	10

Table 10: $N = 1, p(1)$ odd, $a_1 = -1, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
13	5 h, 18 m, 48 s	950202957	5391690	15
11	7 h, 47 m, 53 s	1556110305	7643725	18
9	11 h, 24 m, 5 s	2493608996	12732997	33
7	15 h, 55 m, 38 s	3556887375	19429002	32
5	17 h, 41 m, 50 s	3785354697	22688277	47
3	18 h, 14 m, 38 s	3828868936	23761422	37
1	17 h, 15 m, 47 s	3832854314	24724536	49
-1	17 h, 0 m, 56 s	3823220237	24300024	41
-3	16 h, 21 m, 36 s	3764228299	23444963	48
-5	15 h, 9 m, 18 s	3618172259	21028595	40
-7	13 h, 26 m, 19 s	3274835793	17780238	35
-9	9 h, 46 m, 59 s	2773480926	13767779	29
-11	8 h, 6 m, 39 s	2099537336	9657739	21

Table 11: $N = 1, p(1)$ odd, $a_1 = -2, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
12	6 h, 13 m, 29 s	1239519400	2980012	20
10	9 h, 8 m, 9 s	1957520994	4985198	26
8	14 h, 0 m, 24 s	3123125685	8992737	31
6	16 h, 4 m, 36 s	3616843593	11059253	37
4	16 h, 54 m, 51 s	3825661728	12173850	36
2	16 h, 51 m, 16 s	3735947301	12344096	30
0	17 h, 14 m, 42 s	3825543375	13173928	35
-2	16 h, 39 m, 11 s	3674298427	12947661	30
-4	16 h, 14 m, 53 s	3643262296	13031280	25
-6	14 h, 43 m, 25 s	3281758174	11643263	21
-8	13 h, 34 m, 28 s	2978565941	10159860	13
-10	12 h, 19 m, 17 s	2416937863	7810868	17

Table 12: $N = 1, p(1)$ odd, $a_1 = -2, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
12	7 h, 23 m, 45 s	1233930720	5655239	18
10	11 h, 12 m, 1 s	1952394908	9124727	23
8	15 h, 27 m, 33 s	3121233139	15820322	33
6	17 h, 25 m, 25 s	3614703383	19917140	43
4	16 h, 27 m, 54 s	3823031427	22009048	37
2	20 h, 1 m, 30 s	3733483113	22361141	38
0	20 h, 19 m, 33 s	3822843586	23333309	44
-2	19 h, 40 m, 15 s	3671777272	22416200	47
-4	19 h, 7 m, 2 s	3641792420	21832704	34
-6	17 h, 7 m, 2 s	3641792420	21832704	34
-8	16 h, 5 m, 2 s	2981353759	16229576	40
-10	13 h, 45 m, 23 s	2418670854	12033190	30

Table 13: $N = 1, p(1)$ odd, $a_1 = -3, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
13	10 h, 3 m, 3 s	1269199540	3023598	13
11	13 h, 57 m, 20 s	1959314162	4854507	13
9	19 h, 17 m, 11 s	3091275511	7931980	31
7	24 h, 6 m, 31 s	4300233823	11250694	33
5	25 h, 31 m, 30 s	4673327487	12698948	32
3	25 h, 9 m, 57 s	4748218467	13272736	36
1	25 h, 21 m, 57 s	4658106986	13890402	37
-1	25 h, 20 m, 50 s	4464518947	14003141	26
-3	24 h, 56 m, 11 s	4185895829	13907828	30
-5	24 h, 29 m, 51 s	3834723637	13009017	23
-7	23 h, 31 m, 49 s	3422501725	11637493	17
-9	22 h, 40 m, 29 s	3021506300	9773606	12
-11	21 h, 16 m, 16 s	2564976695	7528537	5

Table 14: $N = 1, p(1)$ odd, $a_1 = -3, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
13	11 h, 13 m, 27 s	1264947933	4573313	15
11	15 h, 54 m, 33 s	1952998953	7389536	18
9	22 h, 29 m, 52 s	3085567875	13816914	23
7	33 h, 29 m, 46 s	4296889053	20951361	42
5	35 h, 12 m, 32 s	4668409371	24454171	46
3	35 h, 35 m, 12 s	4742328112	26063227	46
1	35 h, 44 m, 34 s	4653043055	27819797	47
-1	35 h, 15 m, 47 s	4460759238	27925149	56
-3	34 h, 22 m, 33 s	4182939868	27746647	46
-5	33 h, 18 m, 58 s	3833535936	25804470	42
-7	31 h, 5 m, 8 s	3421965105	22880311	43
-9	27 h, 50 m, 0 s	3021688014	18694662	25
-11	24 h, 35 m, 49 s	2564837262	13693046	18

Table 15: $N = 1, p(1)$ odd, $a_1 = -4, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
14	25 h, 53 m, 5 s	1770114944	2917701	11
12	30 h, 9 m, 5 s	2413918068	4051358	18
10	39 h, 47 m, 19 s	3808161747	6174230	17
8	55 h, 6 m, 37 s	5622079701	8812898	24
6	63 h, 41 m, 30 s	6685243469	10601811	30
4	62 h, 16 m, 55 s	6701517494	11153786	37
2	64 h, 58 m, 58 s	6713518838	11534049	31
0	61 h, 50 m, 58 s	6305445484	11411130	37
-2	63 h, 43 m, 30 s	6042332433	11279919	28
-4	60 h, 43 m, 18 s	5466644280	10606410	23
-6	60 h, 3 m, 59 s	5047862607	9849417	24
-8	62 h, 25 m, 42 s	4448941128	8595628	17
-10	61 h, 45 m, 49 s	3997796362	7081048	8

Table 16: $N = 1, p(1)$ odd, $a_1 = -4, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
14	26 h, 33 m, 10 s	1767371292	3261990	13
12	32 h, 31 m, 33 s	2410544431	5311697	17
10	45 h, 52 m, 16 s	3804347753	10410880	20
8	63 h, 46 m, 43 s	5619726696	18274683	43
6	74 h, 22 m, 24 s	6682574635	23255389	45
4	74 h, 33 m, 28 s	6696916509	26100742	50
2	70 h, 14 m, 33 s	6709680737	28628511	47
0	65 h, 6 m, 42 s	6303019661	29547537	51
-2	61 h, 55 m, 10 s	6041344065	30049853	10
-4	59 h, 20 m, 36 s	5466465207	28670509	43
-6	58 h, 34 m, 2 s	5048210948	26522248	37
-8	43 h, 59 m, 38 s	4448857827	22645529	36
-10	38 h, 2 m, 51 s	3995990083	18105752	20

Table 17: $N = 1, p(1)$ even, $a_1 = 0, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
12	0 h, 37 m, 12 s	180981151	4641	0
10	0 h, 56 m, 54 s	303920608	7069	0
8	1 h, 27 m, 4 s	4292097044	12068	0
6	1 h, 49 m, 4 s	623716942	15535	0
4	1 h, 50 m, 0 s	637577825	17437	0
2	1 h, 56 m, 28 s	669340606	18660	0
0	1 h, 54 m, 27 s	650072123	18546	0
-2	1 h, 57 m, 14 s	669757717	19321	0
-4	1 h, 46 m, 53 s	627773919	18178	0
-6	1 h, 46 m, 1 s	59868848	17113	1
-8	1 h, 41 m, 15 s	506084734	14211	2
-10	1 h, 35 m, 35 s	424585186	11356	2
-12	1 h, 21 m, 13 s	317543910	7894	0

Table 18: $N = 1, p(1)$ even, $a_1 = 0, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
12	0 h, 42 m, 39 s	181133619	4530	0
10	1 h, 7 m, 1 s	304409338	7504	0
8	1 h, 39 m, 9 s	492215007	12273	0
6	2 h, 1 m, 19 s	623768384	16857	1
4	1 h, 59 m, 7 s	638077079	17958	0
2	1 h, 58 m, 25 s	669979397	18965	0
0	1 h, 50 m, 27 s	650576166	18369	0
-2	1 h, 50 m, 27 s	650576166	18369	2
-4	1 h, 44 m, 13 s	627707321	16435	0
-6	1 h, 43 m, 36 s	598237613	14133	0
-8	1 h, 30 m, 17 s	505422221	11407	2
-10	1 h, 22 m, 31 s	423969933	7989	0
-12	1 h, 11 m, 33 s	317170785	5500	0

Table 19: $N = 1, p(1)$ even, $a_1 = -1, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
13	1 h, 14 m, 43 s	245056824	3289	0
11	1 h, 41 m, 42 s	400832110	5213	1
9	2 h, 34 m, 51 s	641745954	8448	0
7	3 h, 40 m, 7 s	915053718	16400	0
5	3 h, 53 m, 58 s	974040284	17183	0
3	3 h, 55 m, 23 s	985304904	19818	0
1	3 h, 55 m, 58 s	986380297	19101	0
-1	3 h, 52 m, 23 s	983701867	22014	2
-3	3 h, 47 m, 55 s	967948644	20122	1
-5	3 h, 39 m, 19 s	929569753	21246	3
-7	3 h, 26 m, 51 s	840245421	15930	1
-9	3 h, 12 m, 52 s	71125794	14888	0
-11	2 h, 56 m, 53 s	538842873	9462	0

Table 20: $N = 1, p(1)$ even, $a_1 = -1, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
13	1 h, 17 m, 1 s	244611064	8057	0
11	1 h, 46 m, 52 s	400260951	11869	0
9	2 h, 41 m, 54 s	641415737	18317	0
7	3 h, 45 m, 39 s	915049909	30517	0
5	4 h, 0 m, 10 s	974009096	33854	0
3	4 h, 2 m, 59 s	985323271	37540	2
1	4 h, 1 m, 59 s	986419342	37136	0
-1	3 h, 22 m, 46 s	983814157	38968	1
-3	3 h, 5 m, 6 s	968346418	25850	0
-5	3 h, 38 m, 38 s	930385463	35297	2
-7	3 h, 19 m, 33 s	841449497	28534	0
-9	3 h, 2 m, 27 s	712486628	24329	1
-11	2 h, 53 m, 1 s	539687890	15396	0

Table 21: $N = 1, p(1)$ even, $a_1 = -2, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
12	2 h, 9 m, 39 s	317413108	5655	0
10	3 h, 5 m, 32 s	501792213	1033	0
8	4 h, 44 m, 3 s	801565312	17442	0
6	5 h, 26 m, 21 s	928347526	22971	0
4	5 h, 35 m, 43 s	981955439	23432	0
2	5 h, 30 m, 43 s	958984102	26054	1
0	5 h, 34 m, 39 s	981941771	26167	1
-2	5 h, 22 m, 57 s	943092422	283851	0
-4	5 h, 23 m, 7 s	935193850	27016	1
-6	5 h, 4 m, 29 s	842599656	26194	2
-8	4 h, 49 m, 46 s	765071591	21484	1
-10	4 h, 31 m, 22 s	620688955	18116	0

Table 22: $N = 1, p(1)$ even, $a_1 = -2, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
12	2 h, 13 m, 1 s	315944916	13316	0
10	3 h, 9 m, 54 s	500418841	23190	1
8	4 h, 41 m, 27 s	801011779	37258	0
6	5 h, 23 m, 31 s	927741101	50810	0
4	5 h, 35 m, 41 s	981239279	51034	0
2	5 h, 31 m, 4 s	958325310	57759	1
0	5 h, 33 m, 6 s	981217781	56120	0
-2	5 h, 25 m, 7 s	942403257	59148	0
-4	5 h, 22 m, 4 s	934749306	53626	0
-6	5 h, 2 m, 4 s	842635024	51157	1
-8	4 h, 54 m, 26 s	765708280	41584	0
-10	4 h, 36 m, 21 s	621056508	33142	0

Table 23: $N = 1, p(1)$ even, $a_1 = -3, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
13	4 h, 21 m, 22 s	328683178	8334	0
11	5 h, 55 m, 9 s	507527427	14491	1
9	8 h, 58 m, 5 s	801177265	21270	0
7	12 h, 9 m, 52 s	1115365637	31847	1
5	12 h, 37 m, 44 s	1211890925	33224	1
3	12 h, 39 m, 50 s	1231298356	37086	0
1	12 h, 35 m, 20 s	1207979608	36242	0
-1	12 h, 21 m, 22 s	1158012599	39724	1
-3	12 h, 19 m, 48 s	1085735704	37661	0
-5	11 h, 58 m, 6 s	994951283	38210	0
-7	10 h, 17 m, 33 s	887939876	32494	0
-9	10 h, 4 m, 23 s	784126337	29508	0
-11	9 h, 49 m, 31 s	665427059	21914	0

Table 24: $N = 1, p(1)$ even, $a_1 = -3, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
13	4 h, 19 m, 12 s	327563950	17059	0
11	5 h, 51 m, 34 s	505857067	27001	0
9	8 h, 52 m, 58 s	799639423	50158	0
7	11 h, 43 m, 28 s	1114412118	76152	0
5	12 h, 12 m, 39 s	1210538290	89096	0
3	12 h, 10 m, 54 s	1229694740	95104	1
1	12 h, 8 m, 20 s	1206598841	100704	1
-1	11 h, 57 m, 46 s	1156982645	103803	1
-3	11 h, 52 m, 25 s	1084931296	101493	1
-5	11 h, 30 m, 21 s	994601295	97000	0
-7	10 h, 31 m, 2 s	887771189	82367	0
-9	9 h, 58 m, 4 s	784147769	71372	0
-11	9 h, 45 m, 42 s	665355587	49813	0

Table 25: $N = 1, p(1)$ even, $a_1 = -4, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
14	16 h, 6 m, 31 s	473049490	12729	0
12	18 h, 45 m, 10 s	645045967	18323	0
10	25 h, 7 m, 17 s	1017411515	25824	0
8	32 h, 27 m, 2 s	1501594083	39215	0
6	36 h, 18 m, 54 s	1785597226	44311	0
4	36 h, 2 m, 27 s	1790288412	49854	0
2	36 h, 56 m, 10 s	1793441875	48178	0
0	35 h, 19 m, 15 s	1684282951	51381	0
-2	34 h, 2 m, 36 s	1613829211	47633	0
-4	32 h, 34 m, 28 s	1459974737	49038	0
-6	34 h, 12 m, 51 s	1348064219	41976	0
-8	34 h, 26 m, 36 s	1188159088	10370	0
-10	35 h, 44 m, 52 s	1067873319	30257	0

Table 26: $N = 1, p(1)$ even, $a_1 = -4, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
14	12 h, 52 m, 30 s	472323970	10499	0
12	17 h, 55 m, 19 s	644159192	17844	0
10	26 h, 0 m, 21 s	1016393497	31285	0
8	40 h, 40 m, 16 s	1501017244	57368	0
6	44 h, 20 m, 6 s	1784965530	66788	0
4	43 h, 58 m, 43 s	1789127055	79544	0
2	41 h, 41 m, 52 s	1792456576	80997	0
0	40 h, 22 m, 4 s	1683670629	88620	0
-2	40 h, 54 m, 14 s	1613594274	83279	0
-4	39 h, 43 m, 50 s	1459954324	82931	0
-6	40 h, 45 m, 43 s	1348179317	69800	0
-8	29 h, 26 m, 50 s	1188161047	62998	0
-10	29 h, 52 m, 31 s	1067422097	45300	0

Signature (3,3)

Table 27: $N = 1, p(1)$ odd, $a_1 = 0, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
14	55 h	17015964408	45779640	17
12	80 h	30022075232	61032018	44
10	99 h	36237723088	77707862	41
8	120 h	53996268076	117327990	40
6	127 h	61785162223	138516643	30
4	158 h	57701291906	147005961	29
2	164 h	63219360099	156173710	25
0	149 h	62123299154	157407745	26
-2	150 h	63292940569	159883723	25
-4	138 h	61878281673	153280925	29
-6	152 h	62113020140	145335065	14
-8	155 h	58606399906	128557694	7
-10	117 h	55130750026	119915818	4
-12	108 h	47912891080	92121947	3
-14	97 h	41344192605	74619239	1

Table 28: $N = 1, p(1)$ odd, $a_1 = 0, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
14	52 h	20575378882	62392638	14
12	80 h	26618497796	80221561	27
10	124 h	40347460405	110804057	32
8	157 h	58582545203	142407810	38
6	138 h	61794588643	155202485	31
4	145 h	61830457113	146672107	36
2	215 h	63230831374	137597618	22
0	185 h	62131360703	122314500	28
-2	191 h	63298625016	109726743	23
-4	193 h	61879732650	93441580	21
-6	160 h	62105461945	79477910	10
-8	123 h	54374018265	63828674	18
-10	100 h	55096212663	51004031	7
-12	103 h	47875887120	40515250	6
-14	111 h	41313015452	32368372	3

Table 29: $N = 1, p(1)$ odd, $a_1 = -1, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
15	85 h	29095606403	99807888	31
13	120 h	38611699318	127777785	52
11	151 h	50781160750	174145749	63
9	220 h	72734577998	250859359	78
7	245 h	87439454679	292942337	76
5	267 h	88399788207	323180365	79
3	274 h	88417802130	332685782	51
1	275 h	91735747480	317272084	67
-1	271 h	124704524357	306796920	38
-3	268 h	88255909727	294249191	48
-5	252 h	87381267143	270658613	24
-7	231 h	88573154172	350682234	22
-9	215 h	79457871937	204656605	14
-11	196 h	77204082187	173977912	15
-13	192 h	66427919478	139708256	5

Table 30: $N = 1, p(1)$ odd, $a_1 = -1, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
15	114 h	29077911610	99179465	31
13	95 h	51988907711	123722988	54
11	182 h	51608602513	171930667	55
9	286 h	70180008167	229526371	71
7	230 h	86839133156	253826914	56
5	256 h	91714932556	279732771	85
3	252 h	91724941645	251992426	56
1	247 h	91735747480	265396072	63
-1	238 h	91730356718	245664322	50
-3	230 h	91565604647	238878100	48
-5	222 h	90897046131	204702550	32
-7	205 h	88600288625	182841140	26
-9	188 h	84045458498	152092880	8
-11	174 h	77239687368	125631725	10
-13	174 h	66451037133	95267201	3

Table 31: $N = 1, p(1)$ odd, $a_1 = -2, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
16	100 h	26288813372	111905517	20
14	113 h	32016720320	121057800	50
12	150 h	43616621966	166704490	52
10	209 h	55836571696	235694637	67
8	282 h	81063773550	322407478	71
6	321 h	88605033664?	355160899	75
4	327 h	9112941836	366596467	67
2	324 h	89674301116	356497435	64
0	318 h	91285059848	354416496	47
-2	318 h	89222564523	333955534	41
-4	316 h	89771652261	320704593	37
-6	296 h	86383152280	289849920	27
-8	278 h	83988000414	261651424	29
-10	253 h	76938408441	222402892	13
-12	235 h	70146406144	18689279	7
-14	196 h	59430450455	144171160	11

Table 32: $N = 1, p(1)$ odd, $a_1 = -2, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
16	94 h	26224800683	104432524	22
14	119 h	31970067935	131075842	38
12	156 h	44088681659	178223404	46
10	211 h	60402418070	224724733	57
8	270 h	80985624618	283388366	69
6	287 h	88593433768	274588021	79
4	290 h	91117926440	289076983	66
2	287 h	86555547198	273147896	59
0	279 h	91273044433	263608876	47
-2	290 h	89207697297	237761675	33
-4	282 h	89753710941	21766240	25
-6	240 h	86367783982	185353662	23
-8	231 h	84004207741	157760017	11
-10	208 h	76851647775	123356446	8
-12	192 h	69455927505	95140716	6
-14	171 h	59643849841	64189314	3

Table 33: $N = 1, p(1)$ odd, $a_1 = -3, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
15	124 h	28567207507	5050	43
13	158 h	40433919183	3957	50
11	213 h	56362376831	6282	62
9	298 h	78630037106	8358	82
7	350 h	93922206854	6355	67
5	357 h	98398730074	6530	58
3	359 h	98789808403	9151	69
1	406 h	98136982155	4999	56
-1	437 h	92194062297	5705	43
-3	422 h	93244611010	5754	27
-5	405 h	88925795473	3633	36
-7	333 h	80782784505	2886	18
-9	310 h	77529587310	3250	15
-11	340 h	71025607426	1265	8
-13	277 h	6364057276	1142	11

Table 34: $N = 1, p(1)$ odd, $a_1 = -3, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
15	125 h	29854787617	4828	31
13	203 h	40375529767	3448	46
11	208 h	56313791682	5297	58
9	284 h	79207983927	6867	61
7	355 h	93894229325	5332	56
5	339 h	91188000930	5472	51
3	344 h	98749398649	7269	63
1	335 h	96102585537	3890	43
-1	332 h	94261151607	4497	26
-3	314 h	93215701183	4297	29
-5	300 h	89002226922	2630	30
-7	291 h	83765445516	2064	16
-9	271 h	77518125614	2169	15
-11	262 h	70919224320	838	8
-13	248 h	66340066573	634	7

Table 35: $N = 1, p(1)$ odd, $a_1 = -4, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
16	248 h	41262980119	3367	35
14	287 h	51902260298	4376	28
12	391 h	59372936714	5265	75
10	526 h	83244965904	5265	68
8	654 h	108374289655	6001	71
6	782 h	117411732308	6568	59
4	721 h	119151108617	5992	81
2	710 h	111608757159	5711	56
0	691 h	108374289655	5042	73
-2	705 h	103445714410	4327	40
-4	659 h	99334260522	3562	42
-6	650 h	96244597564	2839	27
-8	661 h	89592742944	2087	15
-10	604 h	81654304873	1481	12
-12	613 h	73693483437	939	10

Table 36: $N = 1, p(1)$ odd, $a_1 = -4, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
16	237 h	33145786637	2886	43
14	294 h	42044593144	3809	30
12	382 h	83200045829	4203	51
10	521 h	83140593879	5145	48
8	607 h	108346909124	4993	49
6	676 h	117376838881	5065	58
4	693 h	156394540057	4714	49
2	669 h	114744318133	4394	49
0	668 h	113114459513	3687	49
-2	668 h	106928330483	3326	26
-4	912 h	96753160297	2508	27
-6	725 h	95235088343	1942	15
-8	586 h	89587293574	1538	16
-10	580 h	81649858105	953	5
-12	539 h	75617490742	624	8

Table 37: $N = 1, p(1)$ even, $a_1 = 0, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
14	17 h	5093036962	21	0
12	21 h	6862070214	69	0
10	28 h	9987839257	29	0
8	36 h	13370533269	75	1
6	45 h	15298403391	59	0
4	39 h	15306487920	73	0
2	77 h	15652930467	48	0
0	39 h	15380958920	85	0
-2	64 h	15670269893	24	0
-4	52 h	15319835462	59	2
-6	39 h	15377997425	27	0
-8	39 h	14509072036	34	0
-10	43 h	13646946739	14	0
-12	91 h	11859063158	31	0
-14	47 h	10232980070	3	0

Table 38: $N = 1, p(1)$ even, $a_1 = 0, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
14	18 h	5096983665	15	0
12	21 h	6867117886	61	0
10	36 h	9992655511	29	1
8	90 h	13373033399	64	0
6	45 h	15300627838	53	0
4	44 h	15309300182	68	0
2	44 h	15655703512	28	0
0	58 h	15382898690	46	0
-2	104 h	15671621008	18	0
-4	74 h	15320113657	50	0
-6	54 h	15375969901	36	1
-8	40 h	14503566777	24	0
-10	39 h	13638177444	13	0
-12	79 h	11849732065	19	0
-14	96 h	10225140873	2	0

Table 39: $N = 1, p(1)$ even, $a_1 = -1, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
15	34 h	7362299450	115	0
13	41 h	9772638801	78	0
11	80 h	13067535030	167	0
9	77 h	18414920982	237	0
7	143 h	22138917992	238	0
5	153 h	23217081098	152	1
3	100 h	23224473277	243	0
1	133 h	23227209720	231	1
-1	314 h	23225798547	212	0
-3	100 h	23183767426	130	0
-5	108 h	22962367844	139	1
-7	158 h	22428238666	128	0
-9	93 h	21274756135	107	0
-11	144 h	19551926516	43	0
-13	94 h	16818643596	45	0

Table 40: $N = 1, p(1)$ even, $a_1 = -1, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
15	33 h	7357562246	108	0
13	73 h	9769417571	103	0
11	57 h	13065517469	178	1
9	76 h	18413484262	194	0
7	125 h	22138776036	202	0
5	100 h	23217030066	167	0
3	100 h	22387113455	183	0
1	189 h	23227209720	202	0
-1	135 h	23225864456	156	0
-3	100 h	23184406829	118	1
-5	100 h	22964486908	124	0
-7	113 h	22432982385	109	0
-9	146 h	21282085713	72	0
-11	91 h	19560670883	36	0
-13	128 h	16825607202	32	0

Table 41: $N = 1, p(1)$ even, $a_1 = -2, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
16	65 h	6725582403	65	0
14	55 h	8189909019	43	0
12	67 h	11285305845	80	0
10	97 h	15451110605	63	0
8	140 h	20724347155	129	1
6	204 h	22652784610	80	0
4	229 h	23298374530	115	1
2	186 h	22926267255	81	0
0	130 h	23338284839	106	0
-2	218 h	22811218995	71	0
-4	132 h	22951618925	71	0
-6	194 h	22084610069	50	0
-8	156 h	21475351802	55	0
-10	295 h	19665653073	25	0
-12	243 h	17927761361	27	0
-14	130 h	15240926152	10	0

Table 42: $N = 1, p(1)$ even, $a_1 = -2, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	# fields
16	51 h	6714435168	67	0
14	66 h	8178119899	43	0
12	93 h	11275403715	85	0
10	90 h	15444501962	50	0
8	212 h	20720601429	112	0
6	170 h	22649898110	75	2
4	204 h	23295488515	102	0
2	200 h	22923807891	73	0
0	179 h	23335258055	84	0
-2	201 h	22807483572	46	0
-4	187 h	22947141205	66	0
-6	157 h	22080839980	33	1
-8	153 h	21474597811	37	0
-10	211 h	19669274051	18	1
-12	244 h	17934434667	26	1
-14	129 h	15244584506	3	0

Table 43: $N = 1, p(1)$ even, $a_1 = -3, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
15	203 h	6856769804	145	0
13	116 h	9272659704	93	0
11	259 h	12929827112	151	0
9	204 h	18158342547	270	0
7	258 h	21551444067	198	0
5	259 h	22576668438	173	0
3	302 h	22666289748	243	2
1	341 h	22517299137	170	0
-1	277 h	22094996893	189	1
-3	243 h	21395596559	136	0
-5	250 h	20404733350	101	0
-7	268 h	19225352797	96	0
-9	260 h	17791063493	95	0
-11	242 h	16295059035	31	0
-13	251 h	14604684396	31	0

Table 44: $N = 1, p(1)$ even, $a_1 = -3, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
15	121 h	6846015796	166	0
13	146 h	9259377197	65	0
11	154 h	12918526968	170	0
9	359 h	18151470806	244	0
7	325 h	21544855213	199	0
5	243 h	22567758756	109	0
3	306 h	22657440705	243	0
1	342 h	22509270797	166	1
-1	296 h	22087278009	160	0
-3	296 h	21388819004	104	0
-5	316 h	20398719789	92	1
-7	243 h	19221243662	81	0
-9	487 h	17788317796	64	0
-11	313 h	16293929181	22	0
-13	232 h	14604241346	17	0

Table 45: $N = 1, p(1)$ even, $a_1 = -4, a_9 = 1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
16	236 h	7984846055	68	0
14	244 h	10128629356	50	0
12	359 h	14292208560	125	0
10	465 h	20040884873	55	0
8	531 h	26094613692	123	0
6	625 h	28269606155	67	0
4	612 h	28687767576	107	0
2	594 h	27755117546	67	1
0	510 h	27241585766	101	0
-2	620 h	25750989627	43	0
-4	590 h	24713234278	67	0
-6	588 h	22934288800	27	0
-8	620 h	21573778690	43	0
-10	576 h	19662300856	19	0
-12	648 h	18209618118	32	0

Table 46: $N = 1, p(1)$ even, $a_1 = -4, a_9 = -1$

S_2	time	$\#p(x)$ created	$\#p(x)$ survived	$\#$ fields
16	311 h	7978463006	43	0
14	249 h	10120512516	38	0
12	408 h	14281552316	75	0
10	477 h	20030060252	52	1
8	581 h	26087762248	100	0
6	899 h	28261004687	61	0
4	545 h	28678669774	96	0
2	724 h	27747535540	43	0
0	584 h	27235756565	67	0
-2	578 h	25746672166	25	0
-4	588 h	24710103616	52	0
-6	562 h	22931917563	24	0
-8	619 h	21572433546	34	0
-10	556 h	19661174908	14	0
-12	582 h	18208597787	6	0

Higher values of N

The previous tables were all related to runs of the algorithm where the value for the parameter N was 1. A priori, this is not the only value of N which must be investigated: in fact, Theorem ?? holds for every value N such that $N \leq (T/n)^{n/2}$, where n is the degree of the studied fields and T is the upper bound for the second absolute symmetric function given by Hunter-Pohst-Martinet Theorem and depending on the choice of S_1 . Thus, for every value of S_1 between 0 and 4, we should run the process for every value of N between 1 and $(T/n)^{n/2}$, and every iteration should be divided in four subcases depending on both the sign of $a_8(a_9)$ and the parity value of the generated polynomials..

Fortunately, many of these values of N can be immediately discarded thanks to assuming the local corrections, and this forces N , which is the absolute norm of an HPM-element, to be not an exact multiple of 2, 3, 4 and 5. Thus, the number of checks that one should run is much more than expected, and furthermore the values of N are such that every run gives no number field as output, making thus this process just a security check which gives no fields.

In the following tables, we provide the values of N that must be studied by our programs for every considered signature and for every choice of S_1 between 0 and 4. Together with this, we indicate also the values of the parameter x_0 and x_1 which are needed by the author defined function **hpeqU** in order to work correctly with with the indicated values of N . Given that no number field occurred as an output, we preferred not reporting any other data for this additional runs: we just say that any iteration can last between few seconds and 10 minutes circa, depending on the signature, the value of S_1 and the value of N . Empirically, the more time consuming runs occur when r_1, S_1 and N are quite big at the same time.

S_1	$(T/n)^{n/2}$	N	x_0, x_1
0	5.8255...	\	\
1	6.0634...	\	\
2	6.8210...	\	\
3	8.2387...	7, 8	0, 1.3
4	10.5813...	7, 8, 9	0, 1.3

Table 47: Signature (2, 3)

S_1	$(T/n)^{n/2}$	N	x_0, x_1
0	12.1839...	7, 8, 9, 11	0, 1.3
1	12.5966...	7, 8, 9, 11	0, 1.3
2	13.8979...	7, 8, 9, 11, 13	0, 1.4
3	16.2875...	7, 8, 9, 11, 13, 16	0, 1.4
4	20.1349...	7, 8, 9, 11, 13, 16, 17, 19	0, 1.45

Table 48: Signature (4, 2)

S_1	$(T/n)^{n/2}$	N	x_0, x_1
0	26.1478...	7, 8, 9, 11, 13, 16, 17, 19, 23, 25	0, 1.5
1	26.8780...	7, 8, 9, 11, 13, 16, 17, 19, 23, 25	0, 1.5
2	29.1606...	7, 8, 9, 11, 13, 16, 17, 19, 23, 25, 27, 29	0, 1.52
3	33.2843...	7, 8, 9, 11, 13, 16, 17, 19, 23, 25, 27, 29, 31, 32, 33	0, 1.54
4	39.7737...	7, 8, 9, 11, 13, 16, 17, 19, 23, 25, 27, 29, 31, 32, 33, 37	0, 1.56

Table 49: Signature (6, 1)

S_1	$(T/n)^{n/2}$	N	x_0, x_1
0	6.2782...	\	\
1	6.5135...	\	\
2	7.2605...	7	0, 1.23
3	8.6520...	7, 8	0, 1.23
4	10.9390...	7, 8, 9	0, 1.23

Table 50: Signature (1, 4)

S_1	$(T/n)^{n/2}$	N	x_0, x_1
0	13.1050...	7, 8, 9, 11, 13	0, 1.34
1	13.5211...	7, 8, 9, 11, 13	0, 1.34
2	14.8311...	7, 8, 9, 11, 13	0, 1.34
3	17.2318...	7, 8, 9, 11, 13, 16, 17	0, 1.36
4	21.0887...	7, 8, 9, 11, 13, 16, 17, 19	0, 1.36

Table 51: Signature (3, 3)