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Faraway matter as a possible substitute for dark matter

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Abstract. A review is given of an attempt, made in two recent papers, to estimate the gravitational action of faraway matter on a test particle, in connection with the velocity dispersion in clusters of galaxies and with the rotation curves of spiral galaxies, respectively. Under the assumptions that faraway matter has a fractal distribution and that the gravitational action has a correlation length of the order of some kiloparsec, the gravitational action of faraway matter appears to be sufficient to explain the observations relative to such two phenomena, without invoking any local, dark matter contribution.

1. Introduction

The thesis illustrated in this paper is that the gravitational action of faraway matter may be a substitute for the local action of invisible, transparent (usually called dark) matter, at least in the two cases in which dark matter was first introduced in order to save the phenomena, namely, the velocity dispersion in clusters of galaxies and the rotation curves of spiral galaxies. The main idea underlying such a thesis came to our mind quite occasionally, in connection with one of our main themes of research, foundational features of classical electrodynamics. Indeed, motivated by a critique (Carati & Galgani 2004) of the way in which Planck was dealing with microscopic models of a black body,[1] we were involved in the problem of a microscopic foundation of dispersion of light in crystals[2]. In this connection it occurred to us to understand how dispersion of light in matter is strictly related to the so–called Wheeler–Feynman identity, which was conjectured by such authors (Wheeler & Feynman 1945) and we were able to prove in a simple model (Carati & Galgani 2003, Marino Carati & Galgani 2007). Now, such an identity has an evident global character, inasmuch as it

[1]Planck was considering the elementary model in which one oscillator is acted upon by an external field, and was unable to deal with the full dynamical system of N oscillators with mutual retarded interactions. He assumed all single oscillators to be independent from each other. This fact plagues all of his work, from which one would deduce an emission of radiation proportional to volume rather than to surface.

[2]A full microscopic treatment of the problem is lacking also in the celebrated book of Born and Huang, because radiation reaction force is neglected. Such a force was present in the previous studies of Planck, who actually was the scientist that discovered it. However, he did not recognize the existence of a cancellation which plays a fundamental role and was first proved by Oseen in the year 1916, and later conjectured on general grounds by Wheeler and Feynman.

takes into consideration the retarded and the advanced potentials "created" by all the charges present in the Universe, and states that the sum of the semidifferences of the retarded and of the advanced fields created by all charges exactly vanishes. This introduced us into the frame of concepts of a global character involving the Universe.
Quite naturally we were thus led, especially after conversations with G. Contopoulos and C. Efthymiopoulos, to consider the analogy between the roles far fields play in electrodynamics and in gravitation theory. Obviously, this analogy is evident to everybody, and was particularly pointed out for example by Einstein himself in his Princeton lectures (Einstein 1922), when he was commenting on the fact that the perturbation to the flat metric satisfies the d'Alembert equation, so that even in general relativity one should deal with retarded potentials, as in electrodynamics. He also commented how one might think of implementing in such a way Mach's idea on the role of faraway matter. However, he estimated that the inertial force due to faraway matter was too small. Now, such considerations were made before Hubble's law on galaxies recession was established. In brief, our idea just amounts to implement Einstein's estimate when Hubble's law is assumed as a phenomenological fact.
This idea was implemented in the paper (Carati Cacciatori & Galgani 2008), where it was shown how the gravitational effect of the faraway matter vanishes if the matter is assumed to be uniformly distributed, while it does not if the distribution is assumed to be fractal. In fact the latter hypothesis had been previously suggested by some authors on an observational basis (Sylos Labini *et al.* 1998), although there is an open debate on this point. So, taking a fractal distribution of dimension 2, the force per unit mass on a test particle was shown to have the typical value of $0.2cH_0$, which appears to agree with the observed one (Milgrom 1983, Milgrom & Beckenstein 1987). An application to the velocity dispersion in the Coma cluster of galaxies was also given in that paper.
Finally, in the work (Carati 2011) an estimate of the gravitational effect of faraway matter on the rotation curves of spiral galaxies was given. To this end, a new assumption was needed, namely that of the field due to distant matter should be uncorrelated beyond a length l, which constitutes a free parameter of the theory. With such an assumption is was shown that the theory fits pretty well the observations, with l of the order of 1 kpc. And this, not only for the most common cases in which the rotation curves decay more slowly than expected from the Newtonian action of the local matter, but also for the few cases in which the decay is faster, to which dark matter cannot provide a solution.
So our "theory" is of a conservative character, entirely framed within classical general relativity, the only new idea being that of estimating the gravitational action of matter when the latter is described as constituted of a discrete system of point–galaxies which are assumed on empirical grounds to obey Hubble's law. The computations then show that the relevant contribution comes from the galaxies which are near the border of the visible Universe, which we call here the faraway matter. Actually, their gravitational action turns out to be negligible if matter is assumed to have a homogeneous distribution, while turns out to be of the correct order of magnitude if the distribution is assumed to be fractal of dimension 2. So our "theory" has little to do with other ones, such as for example MOND (Milgrom 1983) which constitutes a kind of "effective" theory, or

TeVeS (Beckenstein 2004) which is one among the theories alternative to classical general relativity.
In the present paper a short review of the works (Carati Cacciatori & Galgani 2008) and (Carati 2011) is given. In section 2 the analogy between far fields in electrodynamics and in gravitation theory is recalled. In section 3 the model is described, and the first significant result is illustrated, namely, a deduction of the FRW metric. Moreover, the Friedmann–Robertson–Walker (FRW) metric is deduced as a mean metric, and an estimate is obtained for an effective local density, which turns out to be five times the observed one. Finally, the applications to velocity dispersion in clusters of galaxies and to rotation curves in spiral galaxies are illustrated in section 4.

2. Far fields in electrodynamics and in gravitation theory

The relevance of faraway matter can be illustrated through the example of the electromagnetic field. As is well known, the electromagnetic field due to a charge e can be split, according to Maxwell's equations, as the sum of two terms:

- The Coulomb field (or near field) $E \simeq \frac{e}{r^2}$
- The far field $E \simeq \frac{ea}{c^2 r}$, where a is the charge acceleration and c the speed of light.

Clearly, the electromagnetic interaction between distant bodies just reduces to the far field, which decreases with the distance much more slowly than the Coulomb one, and manifests itself as a radiation emitted by the source. If one tries to take into account the radiation emitted by all charges present in the Universe, one meets with paradoxes as that of Olbers, which is just due to the radiation emitted by the faraway objects. It is true that modern cosmological theories allow one to escape such a paradox, but at any rate the far away sources still play a role in producing some background field, the $3\ {}^0K$ cosmic background radiation. In a similar way we will try to take into account the background gravitational field due to distant galaxies, estimating its magnitude, and discussing its possible effects.
First of all one has to recall that, in the electromagnetic case, the far field comes into play because the 4–potential A_μ due to a point charge, of position vector $\mathbf{q}$, is a solution of the d'Alembert equation

$$\Box A_\mu = 4\pi e \dot{\mathbf{q}}_\mu \, \delta(\mathbf{x} - \mathbf{q})$$

($\delta()$ being the Dirac delta function), i.e., is a relativistic effect. So, in order to compute the gravitational effects of far away matter, one has to go beyond Newton's theory, and make use of general relativity.
This cannot be done in full generality, as the full problem is intractable. In fact, in computing the field produced by distant sources, one is faced with a non linear coupled problem, namely,

- The gravitational field is a solution of the Einstein equations having as source the energy–momentum tensor corresponding to the galaxies, dealt with as "particles";

- the motion of the particles (and thus the corresponding energy–momentum tensor) is determined by the force field, that they themselves create as sources

3. The model. First result: the effective FRW metric and the effective matter density

The problem is too complicated, and no one is able to say anything definite about it. One can simplify it, as we will do, by making the following assumptions:

1. the motion of the galaxies is assigned, according to the observation (this corresponds in electrodynamics to the antenna problem, in which the currents are assigned)
2. the Einstein field equation is linearized.

The motion of the galaxies (of position vectors $\mathbf{q}_j$) is assigned according to Hubble's Law $\dot{\mathbf{q}}_j = H_0\,\mathbf{q}_j$, where, for the sake of simplicity, Hubble's constant H_0 is taken time–independent. So, if one thinks of the galaxies as point sources, for the energy–momentum tensor one gets the expression

$$T^{\mu\nu} = \sum_{j=1}^{N} \frac{1}{\sqrt{g}}\, \frac{M_j}{\gamma_j}\; \delta(\mathbf{x}-\mathbf{q}_j)\dot{\mathbf{q}}_j^{\mu}\dot{\mathbf{q}}_j^{\nu}$$

where N is the number of galaxies, of mass M_j, and γ_j is the usual Lorentz factor, while g is the modulus of the determinant of the metric. All derivatives are meant with respect to proper time. In this expression one has to think of the positions $\mathbf{q}_j$ of the galaxies as distributed at "random", i.e., the vectors $\mathbf{q}_j$ are random variables, distributed according to some definite law. The galaxy masses M_j too could in principle be thought of as distributed at random, but we will instead take them all equal, just in order to simplify the discussion.
The second step is the linearization of Einstein's equation for the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ about an unperturbed solution $\eta_{\mu\nu}$. For $\eta_{\mu\nu}$ we take the vacuum solution, i.e., the Minkowsky metric, so that the perturbing metric $h_{\mu\nu}$ has to satisfy the equation (with G the gravitational constant)

$$\Box\big[h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\big] = -\frac{16\pi G}{c^4}T_{\mu\nu}\ ,$$

i.e., essentially the d'Alembert equation, with the energy–momentum tensor $T_{\mu\nu}$ as source. Then the gravitational force will contain a far field term as in the electromagnetic case.
However, before addressing this problem in the next section, we will preliminarily look here at the expression for the metric. As solution of the Einstein equations we consider here (as commonly made in electromagnetism) the retarded one. This is given by

$$h_{\mu\nu} = \frac{-2G}{c^4}\ \sum_{j=1}^{N} \frac{M_j}{\gamma_j}\ \frac{2\dot{q}_\mu^{(j)}\dot{q}_\nu^{(j)} - c^2\eta_{\mu\nu}}{|\mathbf{x}-\mathbf{q}_j|}\Bigg|_{t=t_{\mathrm{ret}}}\ , \tag{1}$$

where the time is the retarded one, i.e., $t_{\rm ret} = t - |\mathbf{q}_j - \mathbf{x}|/c$, which is a function of the position $\mathbf{x}$. We note in passing that this is not the unique solution, because for example one could consider the semisum of the retarded and the advanced potentials, as was recently done by Romero and Pérez (Romero & Pérez 2011). Now, with the retarded solution one deals with the observed positions of galaxies at time t, while the advanced solution requires to know their positions in the remote future, which are unknowable. Thus the estimate of the advanced potentials is quite difficult, and we choose the retarded ones.

Now the metric $h_{\mu\nu}$ is a random variable, because such were assumed to be the positions of the source–galaxies. Then, on averaging, one can get the "mean metric", which should give the properties of the metric in the large: the actual value of the metric will "fluctuate" about the mean value, and such a fluctuation turns out to produce the peculiar effects we will describe later.

If one assumes the distribution of galaxies to be isotropic, for the mean metric one gets the expression

$$\langle\, g_{\mu\nu}\, \rangle\, dx^\mu dx^\nu = (1-\alpha-3\beta)\, c^2 dt^2 - (1+\alpha+\beta) dl^2 \ ,$$

where $dl^2 = dx^2 + dy^2 + dz^2$, and

$$\alpha = \frac{2G}{c^2} \,\langle\, \sum_j \frac{M_j}{|\mathbf{q}_j|}\,\rangle \ , \quad \beta \lesssim \frac{4GH_0^2}{3c^4}\,\langle\, \sum_j M_j |\mathbf{q}_j|\,\rangle \ . \tag{2}$$

So the mean metric turns out to be a Friedmann–Robertson–Walker one.

We meet here with a consistency problem, because in a Friedmann–Robertson–Walker metric the Hubble constant is related to the coefficients α and β by the relation

$$H_0 = \frac{1}{2}\frac{d}{dt} \log \frac{1+\alpha+\beta}{1-\alpha-3\beta} \ .$$

So the value of the r.h.s. has to coincide with the value of H_0 we have assumed phenomenologically for the motion of the galaxies.

On the other hand, one also meets with a big difference with respect to the usual treatment in which matter is dealt with as a continuum. Indeed, in our case α and β depend heavily on the distribution of faraway matter (as the expression (2) explicitly shows), while in the continuum approximation they depend only on the local density. In fact the sums in (2) diverge, as one sees for example in the case of a uniform distribution of galaxies, so that the larger contribution comes from the faraway galaxies.

Introduce an effective density of matter $\rho_{\rm eff}$, defined by requiring that one has

$$\langle \sum \frac{M_j}{|\mathbf{q}_j|}\,\rangle \simeq 4\pi\rho_{\rm eff}\ \frac{{R_0}^2}{2} \ , \quad \langle \sum M_j |\mathbf{q}_j|\,\rangle\ \simeq 4\pi\rho_{\rm eff}\frac{{R_0}^4}{4}$$

where R_0 is the "radius of the universe" (or better of our chart of it).

Then one finds that, between effective density and Hubble's constant, one has the relation

$$\rho_{\rm eff} \simeq \frac{1}{4}\,\frac{3H_0^2}{8\pi G} \ .$$

Thus, using the accepted value for H_0, one finds

$$\rho_{\text{eff}} \simeq 5\rho_0 \ ,$$

where ρ_0 is the (estimated) present density of visible matter. So, our model can fit the observations if the contribution of the distant galaxies (in principle divergent, as remarked above) is four times the contribution of local visible matter. This is the first effect in which the contribution of distant objects might replace the contribution of local dark matter.

4. Application to the velocity dispersion in clusters of galaxies, and to the rotation curves in spiral galaxies

To describe the effect faraway matter has on the velocity dispersion in clusters of galaxies and on the rotation curves of spiral galaxies, we have to compute the "gravitational force" due to the distant galaxies.
We recall that the gravitational field affects the motion of a test particle inasmuch as the motion satisfies Lagrange equations with a Lagrangian that involves the metric tensor, namely,

$$L = g_{\mu\nu} \frac{\mathrm{d}x_\mu}{\mathrm{d}\tau} \frac{\mathrm{d}x_\nu}{\mathrm{d}\tau} \ .$$

So, the equations of motion of a test particle, with position vector $\mathbf{x} = (x_1, x_2, x_3)$ have, for small velocities, the form

$$\ddot{x}_k = -\partial_k h_{00} - \frac{1}{2c} \partial_t h_{0k} + \text{smaller terms} \stackrel{\text{def}}{=} f_k \ .$$

One can check that

- $\partial_k h_{00}$ corresponds to the Newtonian force $\simeq 1/r^2$
- $\frac{1}{2c}\partial_t h_{0k}$ corresponds to the far field $\simeq a/r$, a being again the acceleration of the source.

The analogy with electromagnetic theory is thus complete.
From (1), it is possible to estimate the force per unit mass, $\mathbf{f}$, acting on a test particle. The most important term is that given by the far field. Notice that the acceleration of a galaxy can be obtained by differentiating Hubble's law, so that not only velocities, but also accelerations increase linearly with the distance. In such a way one finally gets

$$\mathbf{f} = \frac{4GH_0^2}{c^2} \sum_{j=1}^{N} M_j \frac{\mathbf{q}_j}{|\mathbf{q}_j|} \ . \tag{3}$$

Notice that, again, $\mathbf{f}$ is a random vector because such are the position vectors $\mathbf{q}_j$. So, for what concerns the mean, assuming an isotropic probability distribution of the galaxies one has

$$\langle \ \mathbf{f} \ \rangle \ = 0 \ ,$$

as expected. Now, although having a vanishing mean, $\mathbf{f}$ is not a vanishing quantity at all. In fact, for a random variable X with vanishing mean, one knows that its typical value is given by its standard deviation σ_X, in our case σ_f. At this point, in order to compute the variance σ_f^2 of $\mathbf{f}$, a crucial role is played by the probability distribution of the galaxies. Two cases may be considered.

1. The positions of the galaxies can be assumed to be independent and identically distributed (as for gases). Then $\mathbf{f}$ is the sum of N independent identically distributed random variables, so that its variance σ_f^2 is simply the sum of the variance of each term. In this way one gets

   $$\sigma_f \simeq cH_0/\sqrt{N} \simeq 0 \ ,$$

   i.e., the force due to the faraway matter is negligible.

2. Some authors (Sylos Labini *et al.* 1998) have proposed that the distribution of the galaxies is a fractal, with dimension $D \simeq 2$. This means that the galaxy positions are correlated, so that the computation of σ_f^2 is no more immediate. In any case, σ_f^2 can be estimated numerically by generating samples from a distribution of points with fractal dimension 2. We generated such a distribution using the recursive relation

   $$\mathbf{q}_{j+1} = \mathbf{q}_j + \mathbf{z}$$

   where $\mathbf{z}$ is a random vector with a Gaussian distribution. A numerical estimate of the standard deviation σ_f gives

   $$\sigma_f \simeq 0.2\, cH_0 \ ,$$

   Thus, in the fractal case the contribution of faraway matter is no more negligible. In fact, it is even of the order of magnitude of that ascribed to dark matter (Milgrom 1983, Milgrom & Beckenstein 1987), both in clusters of galaxies and in connection with the rotation curves of spiral galaxies, as will be discussed below.

So we have estimated the size of the force (per unit mass) due to distant matter, under the fractal assumption. However, one has to recall that the relevant force which effectively acts on an object within a system is the tidal one, namely, $\mathbf{f}-\mathbf{f}^*$, where $\mathbf{f}$ is the force acting on the object, and $\mathbf{f}^*$ that acting at the center of mass of the considered system. Indeed, in the presence of a locally constant force field, according to the equivalence principle (think of Einstein's lift example), the locally constant field can be eliminated by a suitable change of coordinates. Now, the relevant force $\mathbf{f} - \mathbf{f}^*$ turns out to be completely different in the two cases, smooth or nonsmooth.
Indeed, let us estimate the variance σ^2 of $\mathbf{f} - \mathbf{f}^*$ in the two cases. In the smooth case one can estimate $\mathbf{f} - \mathbf{f}^*$ by Taylor expansion, getting

$$\sigma^2 \simeq H_0^2 L^2$$

where L is the linear dimension of the system (the cluster of galaxies, or the galaxy). This contribution is found to be totally negligible for the case of the

"Coma" cluster, and also for the galaxies we studied. On the other hand, for the variance of $\mathbf{f} - \mathbf{f}^*$ in general one has

$$\sigma^2 = 2\ \sigma_f^2 - 2C(\mathbf{f}, \mathbf{f}^*)\ ,$$

where

$$C(\mathbf{f}, \mathbf{f}^*) = \langle \mathbf{f} \cdot \mathbf{f}^* \rangle$$

is the correlation of the two considered quantities,, $\langle\rangle$ denoting mean value. Thus, under the nonsmoothness assumption, which means that the correlation of the tow forces $\mathbf{f}$ and $\mathbf{f}^*$ is negligible, the size of the tidal force is just equal to $\sqrt{2}$ the size of the force itself.
Having determined the size of the tidal force under the decorrelation assumption, there remains now the problem of its global effect on the system considered. The most significant cases are those in which the force field acts as a pressure or as a tension, i.e., the cases in which the force field is locally predominantly centripetal or centrifugal, respectively. Actually one expects that, in the Universe, local structures be formed where the far field conspires to produce pressure. Obviously this remark raises a consistency problem for a possible future more complete theory, in which one abandons the simplifying assumption considered in the present "theory", in which the motions of the sources were taken to be assigned (according to the phenomenological Hubble law). At the present level of approximation, we just consider the choice of the direction (pressure or tension) as a free element of the theory, to be determined form observations. Obviously, one expects that in the large majority of cases, a pressure will be found.
We can now come to a description of the results for the clusters of galaxies and for the rotation curves of spiral galaxies.
In order to estimate the contribution to the virial of a cluster of n galaxies (Carati Cacciatori & Galgani 2008), one starts from the virial theorem, and for the variance σ_v^2 of the velocity on gets

$$n\sigma_v^2 \stackrel{\text{def}}{=} \overline{\sum_i v_i^2} = \overline{\sum_i (\mathbf{f}_i - \mathbf{f}^*) \cdot \mathbf{x}_i}\ ,$$

where $\mathbf{f}_i$ is the force (per unit mass) due to the faraway matter acting on the i–th galaxy of the cluster located at $\mathbf{x}_i$, while $\mathbf{f}^*$ is the value of the force field at the center of mass of the cluster.
Assuming that the forces acting on the different galaxies of the cluster be uncorrelated, and moreover that the force field acts as a pressure, which helps stabilizing the cluster[3], one gets

$$\sigma_v^2 \simeq 0.07\ cH_0 L\ . \tag{4}$$

For the Coma cluster (L of the order of 1 Mpc) this formula gives $\sigma_v = 900$ km s^{-1} against an observed value of 700 km s^{-1}. So the contribution of faraway matter can explain the measured value of the variance σ_v^2, without any need of dark matter.

[3]One should recall that the gravitational force due the visible matter of the cluster is utterly unable to keep the cluster together.

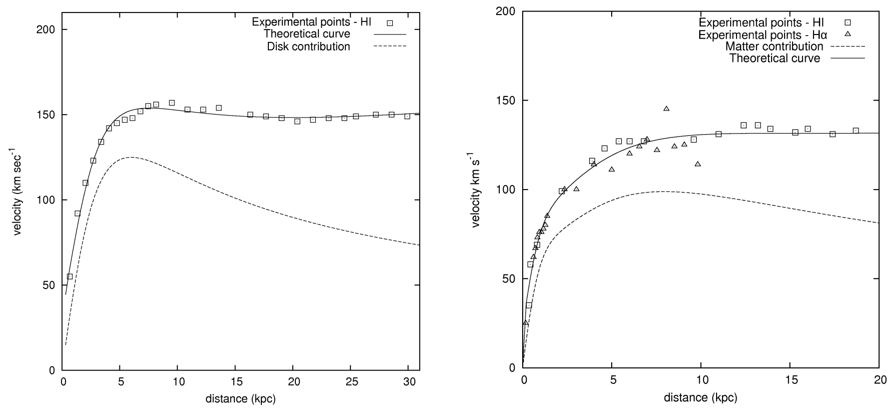

Figure 1. The rotation curves for the galaxy NGC 3198 (left) and NGC 2403 (right). Solid line is the theoretical curve with the contribution of faraway matter taken into account, dashed line refers to the contribution of the local matter.

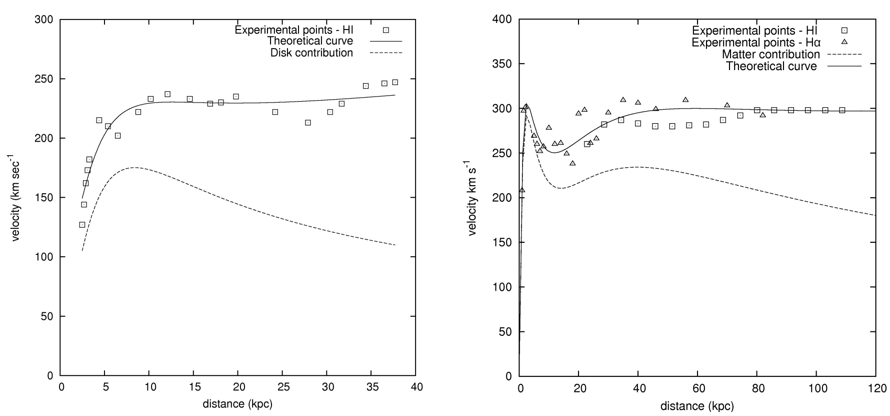

Figure 2. The rotation curves for the galaxy NGC 4725 (left) and UGC 2885 (right). Solid line is the theoretical curve with the contribution of faraway matter taken into account, dashed line refers to the contribution of the local matter.

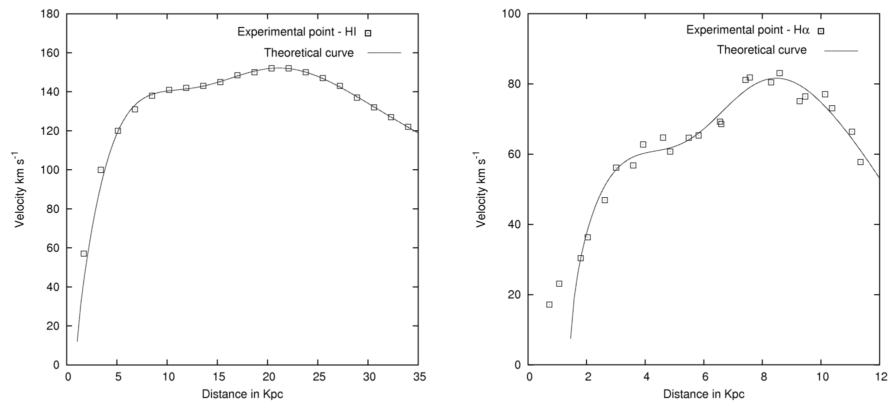

Figure 3. The rotation curves for the galaxy NGC 864 (left) and AGC 400848 (right). Solid line is the theoretical curve with the contribution of faraway matter taken into account. The decrease of the rotation curve is faster then keplerian.

Table 1. Value of the correlation l, for four galaxies.

Galaxy	Mass	Mass/Luminosity	Correlation length
NGC 3198	$4.0\ 10^{10}\ M_\odot$	$4.6\ M_\odot/L_\odot$	0.6 kpc
NGC 2403	$3.5\ 10^{10}\ M_\odot$	$4.4\ M_\odot/L_\odot$	0.8 kpc
UGC 2885	$1.0\ 10^{12}\ M_\odot$	$2.1\ M_\odot/L_\odot$	1.7 kpc
NGC 4725	$1.1\ 10^{11}\ M_\odot$	$2.1\ M_\odot/L_\odot$	3.1 kpc

For what concerns the rotation curves in spiral galaxies (Carati 2011), at variance with the case of the clusters of galaxies one cannot forget the contribution of the local visible matter, which is the larger one. This force can be taken in the nonrelativistic approximation, i.e., expressed in terms of a potential V^{loc}, which depends on the distribution of the local matter.
In order to describe the effects of the force due to the faraway matter in this case, the treatment requires to assume a decorrelation property, i.e., that the correlation decreases exponentially on a certain scale l, which plays the role of a free parameter to be determined by fits with the observations.
In this case, one gets for the speed v of rotation the expression

$$\frac{3}{2}\frac{v^2}{r} = -\partial_r V^{loc}(r) - \partial_r V^{eff} \ , \tag{5}$$

where $\partial_r V^{eff}$ comes from the contribution of the faraway galaxies. This term has to be understood not as a derivative of a potential, but as a random term with vanishing mean, the standard deviation of which can be estimated. One finds

$$\partial_r V^{eff} \simeq \pm 0.2 H_0 c \sqrt{\frac{l}{r}} \ . \tag{6}$$

We still have a free choice for the sign of this term, which entails either a pressure, which helps keeping the galaxy together, or a tension, which tends to break it apart. One can conjecture that the positive sign has to appear more often. Actually, in the literature there are reported observations for a small percentage of galaxies, in which the rotation curves decrease faster than expected from the Newtonian action of the local visible matter, which means either that the galaxy is expanding, or that there is a force acting on the system as a tension.
One can obtain the order of magnitude of the parameter l, by fitting the observation with our formula. We report in Table 1 the value of l obtained by fitting the rotation curves of very different galaxies. One can check that the order of magnitude is always the same, $l \simeq 1$ kpc.

5. Conclusions

So we have shown how the the gravitational action of faraway matter may explain the two classical phenomena for which local dark matter was first introduced. Other phenomena exist which are explained in terms of dark matter, and we hope to tackle them in the future. Obviously, the present "theory" introduces some hypotheses, such as fractal structure of the Universe or decorrelation properties of the gravitational field, as usual with scientific theories.

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