THE BLACK–BODY SPECTRUM, AND THE THEORY OF DYNAMICAL SYSTEMS

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We illustrate what we call the Boltzmann–Jeans effect, namely that the relaxation time to equilibrium for oscillators of a given frequency in general increases exponentially fast with the frequency. This leads one to expect that the blackbody spectrum (divided by the density of modes) might present a plateau in the low frequency region.

1 Introduction

The black–body spectrum is a fundamental object of physics, because it gave rise to quantum mechanics on October 19, 1900, through the discovery of Planck's law. The common opinion is that everything is well established from a theoretical poini of view, and that Planck's law is well confirmed by laboratory experiments. Further confirmation also came from the cosmic background radiation (CBR) because, after some preliminary rather puzzling results, the CBR spectrum was found to fit extremely well Planck's law. So possible deviations of the CBR spectrum from Planck's law would be interpreted as supporting one among different cosmological theories rather than as indicating an actual property of the black-body spectrum itself. We propose instead that some deviations from Planck's law in the low frequency (or high temperatures) region might occur.

In fact, concerning the black body spectrum things are not so simple as usually presumed. The first strange fact is that there are essentially no laboratory experiments after the year 1921, and that the available ones fit Planck's law only within 3 percent (a historical review can be found in the second part of my the paper [1]; see also Crovini *et al.* [2]). The available data turn out to lie in the range $0.1 < x < 10$ in terms of the dimensionless parameter

$$x = \frac{h\nu}{kT} \ ,$$

where h and k are Planck's and Boltzmann's contants, ν and T frequency and absolute temperature respectively. By the way, also the available data for the

CBR spectrum have $x \simeq 0,1$ as a lower limit, and it turns out that the data in the low x region present very huge fluctuations.

We come now to the theoretical reasons that lead us to expect that relevant deviations from Planck's law might be found in the low frequency domain (the Boltzmann–Jeans effect, as we call it). The most striking fact is that such a reasoning is based on completely classical arguments. In our opinion, this is not an indication of a weakness of the argument, but instead a hint that something deep might possibly still be understood in the relations between classical and quantum mechanics. This is supported by the fact that the Boltzmann–Jeans effect is based on a mathematical argument which is extremely general and simple, namely the property that the Fourier transform $\hat{f}(\omega)$ of a real analytic function $f(t)$ of a real variable decreases exponentially fast with the angular frequency ω for large frequencies.

Essentially, the reasoning goes as follows. First of all one reduces the problem to that of the average energy of a single oscillator of given frequency, by the familiar procedure of counting the number of equivalent harmonic oscillators in a given range of frequencies per unit volume. Then one admits, as usually presumed in classical physics, that at any given temperature equipartition should hold for all frequencies (namely that the average energy of an oscillator should be independent of frequency, and equal to kT) if an infinite observation time were available, but one wants to take explicitly into account the corresponding relaxation time, namely the amount of time that is actually required to reach equipartition. This is a problem that was discussed by Boltzmann and Jeans, but is usually not considered. Now, there are very general arguments indicating that, due to the analytical property mentioned above (see Section 2), the energy transfer to the high frequencies is extremely nonuniform with respect to frequency. Indeed it turns out that, if energy is initially given to the low frequencies, in general the transfer to a high frequency ω is exponentially slow, so that the relaxation time increases exponentially fast with the frequency ω. Thus one expects that for any fixed observation time t there will be a cutoff frequency $\bar{\omega}(t)$ such that equipartition (a plateau) will prevail for the lower frequencies, while an exponential tail will occur at the high frequencies; moreover, the equipartition front $\bar{\omega}(t)$ will move at an extremely low rate, and will appear practically as being blocked. This is the essence of the Boltzmann–Jeans effect, which we propose might be observable in the black body spectrum and in the analogous phenomena usually described through Planck's law (specific heats of crystals and of poliatomic molecules, CBR and so on). The advancing of such a plateau was exhibited by numerical computations on a simple one–dimensional model of blackbody about twenty years ago [3], and beautiful indications in the same direction were also given in a very interesting paper written by a group of researchers around G. Parisi. [4]

2 The essence of the analytical argument for the Boltzmann–Jeans effect: the Landau–Teller approximation

To go to the heart of the problem, we consider the simplest example for which the Boltzmann–Jeans effect is exhibited. The model is one–dimensional and consists of two points P and Q on a line: P (which we call the spring) is attracted by a linear spring to a fixed origin O, while Q collides with P through a given smooth (say analytic) potential (for example on can think of a typical molecular potential, or more simply to one decaying exponentially fast). Such a model was studied by Kelvin and Poincaré [6] (see also Poincaré [7]) and later by Landau and Teller [8], and is the simplest one describing the essence of the dynamics of a diatomic gas (see Benettin *et al.*[5]). Denote By x and y the abscissas of the spring P and of the impinging point Q respectively. Then one has a system of two coupled equations for the dynamics, namely Newton's equations with the mutual opposite forces due to the given potential, and rigorous estimates on the exchanges of energies occurring in any single collisions are not obtained in an easy way. But things are much better in the first order approximation, which goes as follows. One considers the case in which the initial energy of the spring is very small so that for the uncoupled system the spring remains very close to the origin, i.e. $|x(t)| \ll 1$ in suitable units. Thus in the mutual force F, which depends on the relative position $y - x$, one can replace in a first approximation the relative position $y - x$ just by y. Consequently Newton's equation for the impinging particle decouples from the other one, becoming a closed equation for y, and is in principle solved by a certain function $y = y(t)$ depending on the initial data. Denoting $F(y(t)/m = f(t)$ (m being the mass of P, which can be taken to be unitary), the equation of motion for the spring P then becomes

$$\ddot{x} + \omega^2 x = f(t)$$

with the given function $f(t)$, and everyone knows (see Landau's book on mechanics) that such an equation is solved by

$$z(t) = e^{i\omega t}\big[z_0 + \int_0^t e^{-i\omega s} f(s) ds\big]$$

in terms of the complex variable $z = \dot{x} + i\omega x$. On the other hand the energy $E = (1/2)(\dot{x}^2 + \omega^2 x^2)$ of the spring is immediately written down in terms of

$z(t)$, and the energy exchange $\Delta E = E(+\infty) - E(-\infty)$ in a single collision turns our to have the simple expression

$$\Delta E = \int_{-\infty}^{+\infty} e^{-i\omega s} F(s) ds \ .$$

Thus in the first approximation the energy that in a single collision the colliding particles gives to a spring of frequency ω is the Fourier transform (evaluated at ω) of an analytic function, which, as mentioned in the introduction, is very well known to decrease exponentially fast with the ferquency ω. It can be shown that the qualitative results of such a first order (or Landau–Teller) approximation hold rigorously for the exact equations (see Carati *et al.*[9] and Benettin *et al.*[10]), and this is the basis for the Boltzmann–Jeans effect. The simplicity of the argument is really impressive, and one hardly could imagine that it might have no actual physical consequences.

3 Possible physical consequences

So our idea should be clear. Following Boltzmann (see Boltzmann[11]) we propose that in the blackbody spectrum and in the specific heat of polyatomic molecules and of crystals one is not actually dealing with equilibrium states, but rather with situations of a sort of metaequilibrium. Such situations should indeed be rather similar to the ones discussed in the frame of glasses or spin glasses, with the familiar phenomena or aging and so on. In fact we have made explicit [12] such an analogy in the case of the Fermi–Pasta–Ulam system (namely a one-dimesional model of a crystal).

What about possible experimental confirmation of our guess? Presently no one, but we hope that something positive might come out if one looks carefully at the problem, knowing taht the effect could exist. One example appears to us promising. We refer to the Johnson effect, concerning the power spectrum of the temperature noise in conductors, which was already indicated to one of us by the late Luigi Crovini several years ago as a promising subject. It is well known that the spectrum should be given by equipartition according to classical mechanics, and this was was well confirmed for high temperatures; one expects instead a Planck spectrum for low temperatures, but this was not observed up to some years ago, as mentioned in the most classical book on noise[13]. In recent years low enough temperatures were investigated [14], and deviations from equipartition were eventually observed, going just in the direction expected according to Planck's law. But, as far as we are able to infer from the quoted paper, it seems that the results actually depend on the observation time, evolving towards equipartition if longer and longer observation times are considered. Figure 17 of the paper Webb *et al.*[14] is the interesting one, where one can see the different results after different observation times (if we interpret well the authors, who refer to results after the third and the fourth demagnetization). The authors also make the following comments: "*We believe that there may be metastable states ... with a spectrum of excitation energies, which gradually decay generating heat. The process may be analogous to the generation of heat in the ortho–para conversion of molecular hydrogen.* This is just what we expect, and is exactly what we find in our numerical simulations on the Fermi–Pasta–Ulam model [12], where a glass–like behavior was exhibited. The high nonuniformity with respect to frequency in the relaxation times to equipartition is fundamental also in connection with sound dispersion [15], and was recently very much discussed in the frame of plasma physics by a group of people around O'Neil (see for example O'Neil *et al.*[16] and Beck *et al.*[17]).

The study of fluctuations in the energy exchanges is particularly important. In a recent work of ours, by the title "Analogue of Planck's formula and effective temperature in classical statistical mechanics far from equilibrium" we have shown how the classical fluctuations, described by a simple formula rediscovered recently by Benettin [18], lead formally, through a procedure introduced by Einstein [19] in his contribution to the 1911 Solvay Conference, to a classical analogue of Planck's formula.

Aknowledgments

The contribution of one of us (A.C.) to the present work was made possible by a grant from Fondazione Cariplo per la Ricerca Scientifica.

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