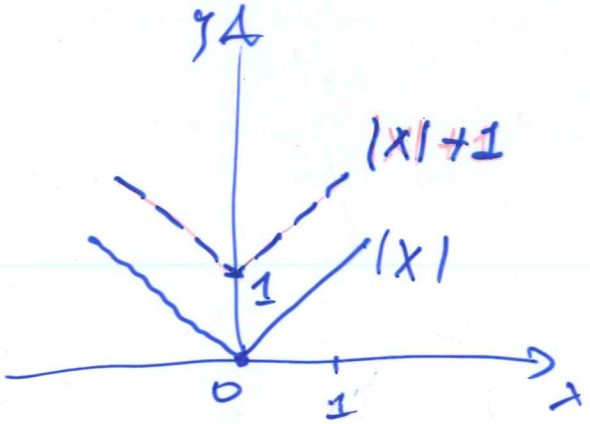
$f(x) = \frac{1}{|x|+1}$. Determinare il grafico

La funzione è così composta:

$$x \xrightarrow{|\ |} |x| \xrightarrow{(\ )+1} |x|+1 \xrightarrow{\frac{1}{(\ )}} f(x)$$

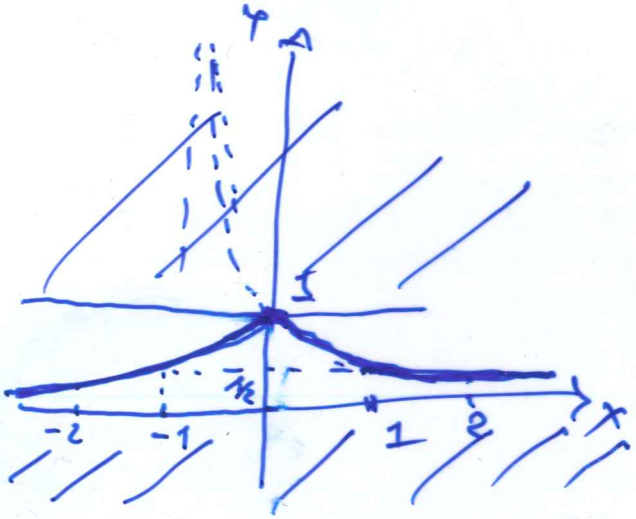


$|x|+1$ è pari

$\Rightarrow$ basta calcolare il grafico di $f(x)$ per $x \geq 0$

$|x|+1 \geq 1 \quad \forall x \in \mathbb{R}$

$\Rightarrow 0 < \frac{1}{|x|+1} \leq 1$



per $x > 0$: $\frac{1}{x+1}$

e per $x < 0$: simmetrico rispetto all'asse y

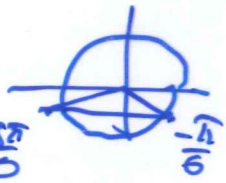
Matematica assistita Arg 2 . 2.10

$2\operatorname{sen} x < 1 \qquad \bigcup_{k\in\mathbb{Z}} \left(\frac{-7\pi}{6} + 2k\pi, \frac{\pi}{6} + 2k\pi\right)$

$2\operatorname{sen} x < -1$

$\operatorname{sen} x = -\frac{1}{2} \iff x = -\frac{\pi}{6} + 2k\pi \quad e \quad x = -\frac{5}{6}\pi + 2k\pi,$

$\Rightarrow \operatorname{sen} x < -\frac{1}{2} \iff \bigcup_{k\in\mathbb{Z}} \left(-\frac{5\pi}{6} + 2k\pi, -\frac{\pi}{6} + 2k\pi\right)$



VERIFICA $-\frac{\pi}{2} \in \left(-\frac{5\pi}{6}, -\frac{\pi}{6}\right)$: $\operatorname{sen}\left(-\frac{\pi}{2}\right) = -1 < \frac{1}{2}$

$\cos x > \frac{\sqrt{3}}{2} \qquad \bigcup_{k\in\mathbb{Z}} \left(-\frac{\pi}{6} + 2k\pi, \frac{\pi}{6} + 2k\pi\right)$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6} + 2k\pi, \quad x = -\frac{\pi}{6} + 2k\pi$

$\tan \frac{x}{2} < \sqrt{3} \qquad \tan \frac{x}{2} = \sqrt{3} \iff$

$\frac{x}{2} = \frac{\pi}{3} + k\pi$

$\Leftarrow$

$\tan \frac{x}{2} < \sqrt{3} \iff -\frac{\pi}{2} + k\pi < \frac{x}{2} < \frac{\pi}{3} + k\pi \iff$

$\iff -\pi + 2k\pi < x < \frac{2\pi}{3} + 2k\pi.$

periodo di $f(x)$ è per definizione il minimo $T \in (0,+\infty)$ t.c. $f(x+T)=f(x)$ $\forall x \in$ I.D.(f)

Se $f(x) = \tan \frac{x}{2}$ $\quad x \xmapsto{(\cdot)/2} \frac{x}{2} \xmapsto{\tan} f(x)$

$$\tan \frac{x+T}{2} = \tan \frac{x}{2}$$

$$\tan \alpha = \tan \beta \iff \alpha - \beta = k\pi \quad k \in \mathbb{Z}$$

Quindi

$$\frac{x+T}{2} - \frac{x}{2} = k\pi \iff \frac{T}{2} = k\pi \iff T = 2k\pi$$

Se minimo $T > 0$ che ha la forma $T = 2k\pi$ l'ottengo per $k=1$:

$$\boxed{T = 2\pi}$$

E la $\tan 2x$? $\quad \frac{\pi}{2}$

Se $\tan \omega x$, ove $\omega \in (0,+\infty)$, il periodo risulta $\frac{\pi}{\omega}$

Risolvere $\operatorname{sen} 3x = \operatorname{sen} 2x$

$\operatorname{sen}\alpha = \operatorname{sen}\beta \iff ?$ $\quad \beta = \alpha + 2k\pi$ oppure

$\beta = \pi - \alpha + 2k\pi \quad k \in \mathbb{Z}$

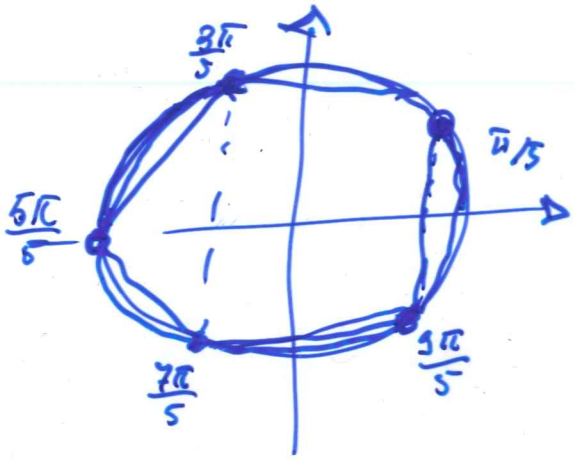
Quindi ci sono due infinità di soluzioni:

$3x = 2x + 2k\pi \implies \boxed{x = 2k\pi \quad k \in \mathbb{Z}}$

oppure

$3x = \pi - 2x + 2k\pi \implies 5x = \pi + 2k\pi$ cioè

$$\boxed{x = \frac{\pi}{5} + k\frac{2\pi}{5} \quad k \in \mathbb{Z}}$$



pentagono regolare!

Risolvere $\operatorname{sen} 3x = \cos 2x$

$\cos\alpha = \operatorname{sen}\left(\alpha + \frac{\pi}{2}\right)$ (il grafico del coseno è traslato di quello di sen. di $\frac{\pi}{2}$ nella direz. di x, verso opposto)

$\operatorname{sen} 3x = \operatorname{sen}\left(2x + \frac{\pi}{2}\right)$ cioè, come sopra, doppia infinità di soluzioni:

$3x = \left(2x + \frac{\pi}{2}\right) + 2k\pi \quad k \in \mathbb{Z}$

$3x = \pi - \left(2x + \frac{\pi}{2}\right) + 2k\pi \quad k \in \mathbb{Z}$

Cioè $\boxed{x = \pi/2 + 2k\pi \quad k \in \mathbb{Z}}$

oppure $\boxed{x = \pi/10 + \frac{2k\pi}{5} \quad k \in \mathbb{Z}}$