Es 35.10 (a)

S46

bis

$$\lim_{n\to+\infty}\left(\operatorname{sen}\frac{1}{n}\right)^{\frac{3}{\ln n}} = [0^0] =$$

$$= \lim_{n\to+\infty} e^{\frac{3}{\ln n}\ln\left(\operatorname{sen}\frac{1}{n}\right)}$$

$$\operatorname{sen}\frac{1}{n} = \frac{1}{n} + o\left(\frac{1}{n}\right) \quad (*)$$

$$\Downarrow \quad \ln\left(\operatorname{sen}\frac{1}{n}\right) = \ln\left(\frac{1}{n} + o\left(\frac{1}{n}\right)\right) \sim \ln\left(\frac{1}{n}\right)$$

$$\Downarrow$$

$$\frac{3}{\ln n}\cdot\ln\left(\operatorname{sen}\frac{1}{n}\right) \sim \frac{3}{\ln n}\cdot\ln\left(\frac{1}{n}\right) = \frac{-3\ln n}{\ln n} \to -3$$

$$\Rightarrow \lim_{n\to+\infty} e^{\frac{3}{\ln n}\ln(\operatorname{sen} 1/n)} = e^{-3}$$

(*) attenzione: che cosa c'è dietro l'asintotico scritto sotto?

$$\ln\left(\frac{1}{n} + o\left(\frac{1}{n}\right)\right) = \ln\left(\frac{1}{n}\left(1 + o\left(\frac{1/n}{1/n}\right)\right)\right) =$$

$$= \ln\left(\frac{1}{n}\left(1 + o(1)\right)\right) =$$

$$= \ln\frac{1}{n} + \ln\left(1 + o(1)\right)$$

Quindi

$$\frac{\ln\left(\frac{1}{n} + o\left(\frac{1}{n}\right)\right)}{\ln\left(\frac{1}{n}\right)} = \frac{\ln\frac{1}{n} + \ln(1+o(1))}{\ln\left(\frac{1}{n}\right)} =$$

$$= \frac{\ln 1/n}{\ln 1/n} + \frac{\ln(1+o(1))}{\ln 1/n} = 1 + o(1) \text{ poiché } \{\ln(1+o(1))\}\to 0 \text{ e } \left\{\ln\frac{1}{n}\right\}\to-\infty$$

e quindi $\displaystyle\lim_{n\to+\infty}\frac{\ln(1/n + o(1/n))}{\ln(1/n)} = 1$, che motiva l'asintotico