$$\lim_{x\to 0} \frac{x(1-\sqrt{1+x}+\sin x)}{1-\cos x} = \left[\frac{0}{0}\right]$$

$$\sqrt{1+x} = (1+x)^{1/2} = 1 + \frac{1}{2}x + o(x)$$

$$\sin x = x + o(x)$$

$$1-\sqrt{1+x}+\operatorname{sen} x = 1 - \left(1 + \frac{1}{2}x + o(x)\right) + x + o(x) =$$

$$= \frac{1}{2}x + o(x)$$

$$\lim_{x\to 0} \frac{x\left(\frac{1}{2}x + o(x)\right)(1+\cos x)}{(1-\cos x)(1+\cos x)} =$$

$$= \lim_{x\to 0} \frac{\left(\frac{1}{2}x^2 + o(x^2)\right)(1+\cos x)}{1-(\cos x)^2} =$$

$$= \lim_{x\to 0} \frac{\frac{1}{\cancel{2}}x^2 \cdot \cancel{2}}{(\operatorname{sen} x)^2} = \lim_{x\to 0} \frac{1}{\left(\frac{\operatorname{sen} x}{x}\right)^2} = 1$$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} \qquad f = \sin x$$

$$\lim_{h\to 0} \frac{\operatorname{sen}(x_0+h) - \operatorname{sen} x_0}{h} = \lim_{h\to 0} \frac{\operatorname{sen} x_0 \cos h + \cos x_0 \operatorname{sen} h - \operatorname{sen} x_0}{h}$$

$$= \lim_{h\to 0} \frac{\operatorname{sen} x_0 \left(\cancel{1} - \frac{h^2}{2} + o(h^2) \cancel{-1}\right) + \cos x_0 \operatorname{sen} h}{h} =$$

$$\boxed{\cos h = 1 - \frac{h^2}{2} + o(h^2)} = \operatorname{sen} x_0 \cdot \underbrace{\left[\lim_{h\to 0}\left(-\frac{h}{2} + o(h)\right)\right]}_{\to 0} + \cos x_0 \underbrace{\left[\lim_{h\to 0} \frac{\operatorname{sen} h}{h}\right]}_{\to 1} =$$

$$= (\operatorname{sen} x_0)\cdot 0 + \cos x_0 \cdot 1 = \cos x_0$$