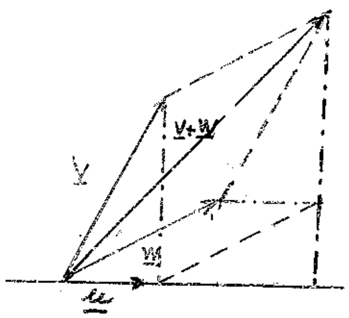
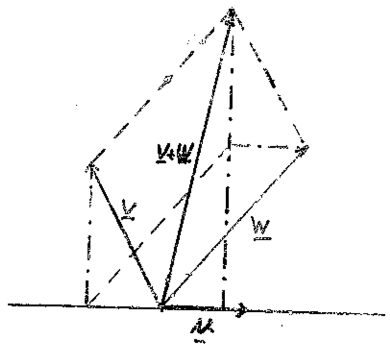
$\circ : V \times V \longrightarrow \mathbb{R}$

Prodotto scalare di due vettori $\underline{v}, \underline{w}$

$\underline{v}\cdot\underline{w} = |\underline{v}|\cdot|\underline{w}|\cos\alpha$ ove $\alpha = \widehat{\underline{v}\,\underline{w}}$, $\alpha \in [0,\pi]$.

- commutativo
- distributivo : $\underline{u}\circ(\underline{v}+\underline{w}) = \underline{u}\circ\underline{v} + \underline{u}\circ\underline{w}$ $\quad \forall \underline{u}, \underline{v}, \underline{w}$





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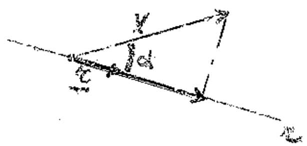
$\cancel{|u|}\,|v+w|\cos(\widehat{u\,(v+w)}) = \cancel{|u|}\cdot|v|\cos(\widehat{u\,v}) + \cancel{|u|}\,|w|\cos(\widehat{u\,w})$

- $\forall t \in \mathbb{R}$, $\forall \underline{v}, \underline{w}$: $(t\underline{v})\circ\underline{w} = t(\underline{v}\circ\underline{w})$

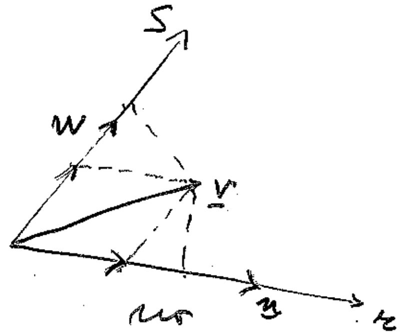
- $\underline{v}\circ\underline{v} = |\underline{v}|\cdot|\underline{v}|\cos 0 = |\underline{v}|^2$
- $\underline{v}\circ\underline{w} = 0$ e $\underline{v}\neq\underline{0}$, $\underline{w}\neq\underline{0} \Longrightarrow \underline{v}\perp\underline{w}$

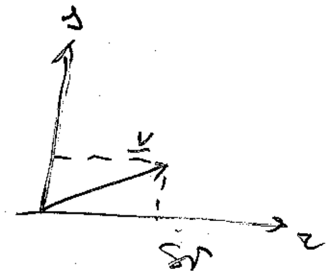
Proiezione ORTOGONALE di un vettore $\underline{v}$ su una retta r (COMPONENTE VETTORIALE di $\underline{v}$)



$$\left[\underline{v}\circ\left(\frac{\underline{r}}{|\underline{r}|}\right)\right]\frac{\underline{r}}{|\underline{r}|}$$

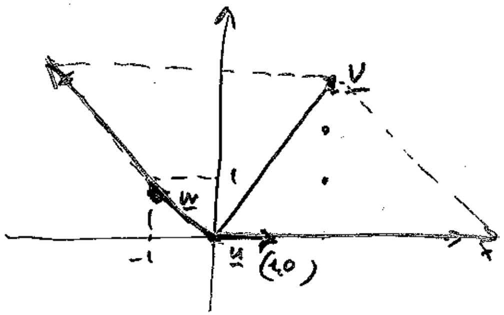
La proiezione ortogonale di un vettore su una retta ha a che vedere con la scomposizione di un vettore lungo due direzioni assegnate?





Problema. Siano $\underline{u} = (1,0)$ e $\underline{w} = (-1,$

Trovare i componenti secondo le direzioni di $\underline{u}$ e $\underline{w}$ del vettore $\underline{v} = (2,3)$.



$\underline{v} = a\,\underline{u} + b\,\underline{w}$

Chi è a?
Chi è b?

$(2,3) = a(1,0) + b(-1,1) =$

$\Rightarrow (2,3) = (a,0) + (-b,b) \Rightarrow (2,3) = (a-b, 0+b) \Longleftrightarrow$

$\Longleftrightarrow \begin{cases} a-b=2 \\ b=3 \end{cases} \qquad \begin{cases} a=5 \\ b=3 \end{cases}$

In termini di componenti come faccio a svolgere un prodotto scalare?

$$\underline{v} = (v_1, v_2) = v_1 \underline{i} + v_2 \underline{j}$$

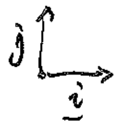
$$\underline{w} = (w_1, w_2) = w_1 \underline{i} + w_2 \underline{j}$$

$$(\underline{v} \cdot \underline{w}) = (v_1 \underline{i} + v_2 \underline{j}) \cdot (w_1 \underline{i} + w_2 \underline{j}) =$$

$$= (v_1 \underline{i} + v_2 \underline{j}) \cdot w_1 \underline{i} + (v_1 \underline{i} + v_2 \underline{j}) \cdot w_2 \underline{j} =$$

$$= v_1 \underline{i} \cdot w_1 \underline{i} + v_2 \underline{j} \cdot w_1 \underline{i} + v_1 \underline{i} \cdot w_2 \underline{j} + v_2 \underline{j} \cdot w_2 \underline{j} =$$

$$= v_1 w_1 \underbrace{\underline{i} \cdot \underline{i}}_{1^2} + v_2 w_1 \underbrace{\underline{j} \cdot \underline{i}}_{0} + v_1 w_2 \underbrace{\underline{i} \cdot \underline{j}}_{0} + v_2 w_2 \underbrace{\underline{j} \cdot \underline{j}}_{1^2}$$



$$= v_1 w_1 + v_2 w_2$$

$$\underline{v} = (v_1, v_2, v_3) \qquad \underline{w} = (w_1, w_2, w_3)$$

$$\boxed{\underline{v} \cdot \underline{w} = v_1 w_1 + v_2 w_2 + v_3 w_3 = |\underline{v}|\,|\underline{w}| \cos\theta}$$

Trovare l'angolo tra i due vettori

$\underline{v} = (1,2,3)$ e $\underline{w} = (3,-1,2)$

$$\cos\theta = \frac{\underline{v} \cdot \underline{w}}{|\underline{v}|\,|\underline{w}|} = \frac{1 \cdot 3 + 2(-1) + 3 \cdot 2}{\sqrt{1^2+2^2+3^2}\,\sqrt{3^2+1^2+2^2}} = \frac{7}{\sqrt{14}\sqrt{14}} =$$

$$= \frac{7}{14} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Trovare un vettore ortogonale a $\underline{v} = (-1,2,1)$ e a $\underline{w} = (3,1,4)$ di modulo 1

$$\underline{u} = (x,y,z)$$

$$\begin{cases} \underline{u} \cdot \underline{v} = 0 \\ \underline{u} \cdot \underline{w} = 0 \\ |\underline{u}| = 1 \end{cases} \quad \begin{cases} (x,y,z) \cdot (-1,2,1) = 0 \\ (x,y,z) \cdot (3,1,4) = 0 \\ x^2+y^2+z^2 = 1 \end{cases} \quad \begin{cases} -x+2y+z=0 \\ 3x+y+4z=0 \\ x^2+y^2+z^2=1 \end{cases}$$

$$\begin{cases} x = 2y+z \\ 6y+3z+y+4z=0 \\ x^2+y^2+z^2=1 \end{cases} \quad \begin{cases} x = 2y+z \\ 7y+7z=0 \\ \dots \end{cases} \quad \begin{cases} x = 2y+z \\ z = -y \end{cases}$$

$$\begin{cases} x = 2t - t = t \\ y = t \\ z = -t \\ x^2+y^2+z^2 = 1 \end{cases} \qquad t^2+t^2+t^2 = 1 \qquad t = \pm\frac{1}{\sqrt{3}}$$

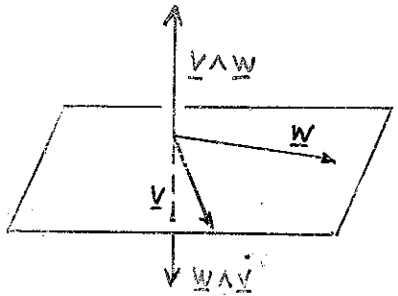
$$\underline{u}_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) \qquad \underline{u}_2 = \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$\boxed{\underline{u}, \underline{v}, \underline{w} \text{ sono indipendenti?}}$

SI

Prodotto vettoriale di $\underline{v}$ e $\underline{w}$ nello spazio di dim. 3

- $|\underline{v} \wedge \underline{w}| = |\underline{v}| \cdot |\underline{w}| \sin\alpha$
- $\underline{v} \wedge \underline{w}$ ortogonale tanto a $\underline{v}$ che a $\underline{w}$
- $\underline{v}, \underline{w}, \underline{v} \wedge \underline{w}$ è una terna DESTRORSA



- anticommutativo
  $\underline{w} \wedge \underline{v} = - \underline{v} \wedge \underline{w}$
- $(t\underline{v}) \wedge \underline{w} = t(\underline{v} \wedge \underline{w})$
- $\underline{v} \wedge \underline{v} = \underline{0}$
- distributivo
  $\underline{u} \wedge (\underline{v} + \underline{w}) = \underline{u} \wedge \underline{v} + \underline{u} \wedge \underline{w}$

Geometricamente $|\underline{v} \wedge \underline{w}| =$ VEDI PAG. SUCCESSIVA.

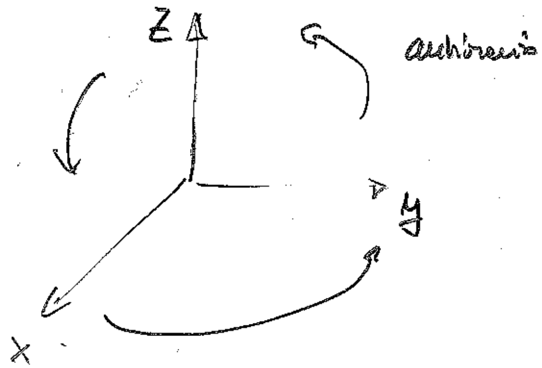
$\underline{i} \wedge \underline{j} = \underline{k}$, $\underline{j} \wedge \underline{k} = \underline{i}$, $\underline{k} \wedge \underline{i} = \underline{j}$ .....

Dunque se $\underline{v} = (v_1, v_2, v_3)$, $\underline{w} = (w_1, w_2, w_2)$

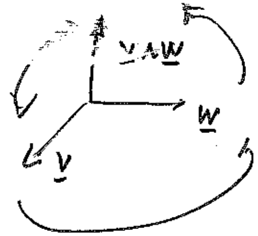
$\underline{v} \wedge \underline{w} = (v_1 \underline{i} + v_2 \underline{j} + v_3 \underline{k}) \wedge (w_1 \underline{i} + w_2 \underline{j} + w_3 \underline{k}) =$ DISTR.

$=$

$\boxed{\text{Es 2 pag 5}}$ $\quad \underline{u} = \dfrac{\underline{v} \wedge \underline{w}}{|\underline{v} \wedge \underline{w}|}$ oppure il suo opposto



x, y, z è una terna destrorsa

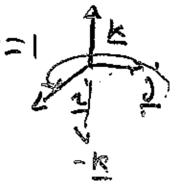


$\underline{i} \wedge \underline{i} = \underline{0}$

$\underline{j} \wedge \underline{j} = \underline{0}$

$\underline{k} \wedge \underline{k} = \underline{0}$

$|\underline{i} \wedge \underline{j}| = 1 \cdot 1 \cdot \operatorname{sen}\frac{\pi}{2} = 1$

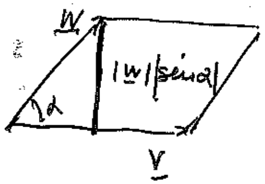


$\underline{i} \wedge \underline{j} = \underline{k}$

$\underline{j} \wedge \underline{i} = -\underline{k}$

$\underline{j} \wedge \underline{k} = \underline{i} \Rightarrow \underline{k} \wedge \underline{j} = -\underline{i}$

$\underline{k} \wedge \underline{i} = \underline{j} \Rightarrow \underline{i} \wedge \underline{k} = -\underline{j}$



$|\underline{v} \wedge \underline{w}|$ = area del parallelogr. di lati $|\underline{v}|$ e $|\underline{w}|$ con angolo compreso α

$$\underline{v} = v_1 \underline{i} + v_2 \underline{j} + v_3 \underline{k} \qquad \underline{w} = w_1 \underline{i} + w_2 \underline{j} + w_3 \underline{k}$$

$\underline{v} \wedge \underline{w} =$ applico la propr. distributiva e quella di omogeneità $\left((t\underline{v}) \wedge \underline{w} = t(\underline{v} \wedge \underline{w}) \right)$

$$= v_1 w_1\, \underline{i} \wedge \underline{i} + v_1 w_2\, \overset{\underline{k}}{\underline{i} \wedge \underline{j}} + v_1 w_3\, \overset{-\underline{j}}{\underline{i} \wedge \underline{k}} +$$

$$+ v_2 w_1\, \overset{-\underline{k}}{\underline{j} \wedge \underline{i}} + v_2 w_2\, \underline{j} \wedge \underline{j} + v_2 w_3\, \overset{\underline{i}}{\underline{j} \wedge \underline{k}} +$$

$$+ v_3 w_1\, \underset{\underline{j}}{\underline{k} \wedge \underline{i}} + v_3 w_2\, \underset{-\underline{i}}{\underline{k} \wedge \underline{j}} + v_3 w_3\, \underline{k} \wedge \underline{k}$$

sono tutti nulli

$$= (v_2 w_3 - v_3 w_2)\, \underline{i} + (v_3 w_1 - v_1 w_3)\, \underline{j} + (v_1 w_2 - v_2 w_1)\, \underline{k} =$$

$$= \begin{array}{cl} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} & \begin{array}{l} \\ \leftarrow \underline{v} \\ \leftarrow \underline{w} \end{array} \end{array}$$