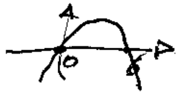
1

$$f(x) = \ln(6x - x^2) + \frac{1}{4}x - \frac{3}{4}$$

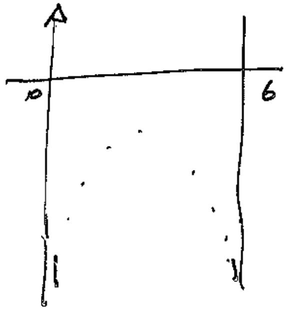


I.D. $6x - x^2 > 0$

$x \in (0, 6)$

$$\lim_{x \to 0^+} f(x) = \left(\lim_{x \to 0^+} \ln[x(6-x)]\right) - \frac{3}{4} = -\infty$$

$$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^-} \ln(x(6-x)) + \frac{3}{4} = -\infty$$



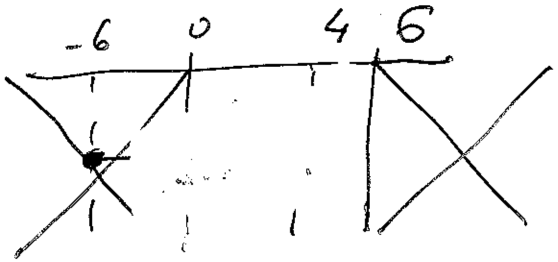
$$f'(x) = \frac{1}{6x - x^2} \cdot (6 - 2x) + \frac{1}{4}$$

$$= \frac{(6-2x) \cdot 4 + 6x - x^2}{4(6x - x^2)} \geq 0$$

in $(0,6)$ il denom è >0 per ipotesi

$$f'(x) > 0 \iff \begin{cases} x \in (0,6) \\ -x^2 - 2x + 24 \geq 0 \end{cases}$$

$$\iff \begin{cases} x \in (0,6) \\ x^2 + 2x - 24 \leq 0 \end{cases} \iff \begin{cases} x \in (0,6) \\ (x+6)(x-4) \leq 0 \end{cases} \iff \begin{cases} x \in (0,6) \\ -6 \leq x \leq 4 \end{cases}$$



$N \geq 0$

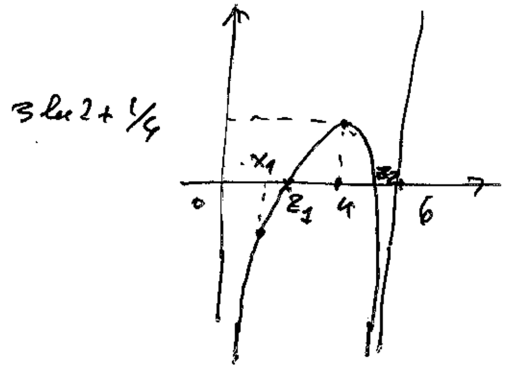
$f'(x) > 0$ se $x \in (0, 4)$ $\Rightarrow$ in $(0,4)$ $f(x)$ cresce

$f'(x) < 0$ se $\in (4, 6)$ $\Rightarrow$ in $(4,6)$ $f(x)$ decresce

$\Rightarrow$ $x = 4$ è un pto di MAX relativo che è anche di max assoluto

2

$$f(4) = \ln(24 - 16) + 1 - \frac{3}{4} = 3\ln 2 + \frac{1}{4}$$



$\Rightarrow$ esiste uno zero in $(0,4)$ poiché $f(4) > 0$ mentre $\lim_{x \to 0^+} f(x) = -\infty$ e quindi esiste un $x_1 \in (0,4)$ in cui $f(x_1) < 0$

Applico il teor degli zeri (in quanto in $[x_1, 4]$ la funz. continua) $\Rightarrow$ esiste almeno uno zero z_1 in $[x_1, 4]$

e 1 solo poiché $f(x)$ cresce quindi è invertibile (biunivoca) e quindi c'è un solo punto in $[x_1, 4]$ tale che $f(x_1) = 0$

Analogamente tra 4 e 6 ci sarà un (solo) altro zero (f qui è monotona decrescente)

tg in $(1, f(1))$ e $(5, f(5))$

$$f(1) = \ln(6-1) + \frac{1}{4} - \frac{3}{4} = \ln 5 - \frac{1}{2} > 0$$

$$f'(1) = \frac{6-2}{6-1} + \frac{1}{4} = \frac{4}{5} + \frac{1}{4} = \frac{21}{20}$$

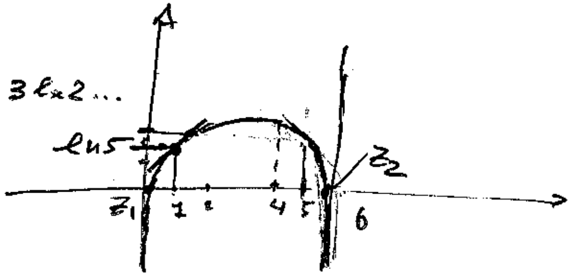
eq. retta tang. in $(1, f(1))$:

$$y - \left(\ln 5 - \frac{1}{2}\right) = \frac{21}{20}(x - 1)$$

(3

$$f(5) = \ln(30-25) + \frac{5}{4} - \frac{3}{4} = \ln 5 + \frac{1}{2}$$

$$f'(5) = \frac{6-10}{30-25} + \frac{1}{4} = \frac{-4}{5} + \frac{1}{4} = \frac{-11}{20}$$



$f(1) > 0$, $f(5) > 0$ $\Rightarrow$ $z_1 \in (0,1]$

$z_2 \in [5,6)$

$$\int \sqrt{2x+1} \cdot \frac{1}{x}\, dx$$

$\boxed{2x+1 = t^2}$ $\xrightarrow{t>0}$

$x = \frac{t^2-1}{2}$

$dx = \frac{1}{2}(2t\,dt) = t\,dt$

$$\int t \cdot \frac{1}{\frac{t^2-1}{2}} \cdot t\,dt = \int \frac{2t^2}{t^2-1}\, dt =$$

$$= 2\int \frac{t^2-1+1}{t^2-1}\, dt = 2\int\left(1 + \frac{1}{t^2-1}\right) dt =$$

$$= 2\left[t + \int\left(\frac{1/2}{t-1} + \frac{-1/2}{t+1}\right) dt\right] = 2\left[t + \frac{1}{2}\left(\ln\frac{|t-1|}{|t+1|}\right)\right] + c$$

$$= 2t + \ln\left|\frac{t-1}{t+1}\right| + c = 2\sqrt{2x+1} + \ln\left|\frac{\sqrt{2x+1}-1}{\sqrt{2x+1}+1}\right| + c$$

(4

$$\int\left(e^{2x} - \sqrt{x^2-5}\right) dx =$$

$$= \int e^{2x}\, dx - \int \sqrt{x^2-5}\, dx =$$

$$= \frac{1}{2} e^{2x} - \int \sqrt{x^2-5}\, dx$$

dove? $\boxed{x > \sqrt{5}}$? $x < -\sqrt{5}$?

$$\sqrt{x^2-5} = x + t$$

$$x^2 - 5 = (x+t)^2 \Rightarrow \cancel{x^2} - 5 = \cancel{x^2} + 2tx + t^2$$

$$\Rightarrow x = \frac{-t^2-5}{2t} \Rightarrow dx = -\frac{2t\cdot t - (t^2+5)}{2t^2}\, dt$$

$$= -\frac{1}{2}\cdot\left(\frac{t^2+5}{t}\right) \qquad = -\frac{t^2-5}{2t^2}\, dt$$

$$\int \sqrt{x^2-5}\, dx = \int\left(-\frac{(t^2+5)}{2t} + t\right)\cdot\frac{-(t^2-5)}{2t^2}\, dt$$

$$= \int \frac{t^2-5}{2t}\cdot\frac{5-t^2}{2t^2}\, dt =$$

$$= -\int \frac{t^4 - 10t^2 + 25}{4t^3}\, dt = -\int\left(\frac{t}{4} - \frac{5}{2}\frac{1}{t} + \frac{25}{4}\frac{1}{t^3}\right) dt$$

.... $t = \sqrt{x^2-5} - x$ ...

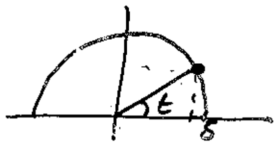
$$\int \sqrt{25-x^2}\, dx$$

per sostituzione

$y = \sqrt{25-x^2}$ ha per grafico una semicirconf ($y \geq 0$) con centro $(0,0)$ e raggio 5

$y = \sqrt{25-x^2}$

$$\begin{cases} x = 5\cos t \\ y = 5 \operatorname{sen} t \end{cases} \quad 0 \le t \le \pi$$



$dx = -5 \operatorname{sen} t \, dt$

$$\int \sqrt{25-x^2}\,dx = \int 5 \operatorname{sen} t \cdot (-5 \operatorname{sen} t)\,dt$$

$\sqrt{25 \operatorname{sen}^2 t} = 5|\operatorname{sen} t| \overset{?}{=} 5 \operatorname{sen} t$

sì perché $0 \le t \le \pi$

$$= \boxed{25 \int -\operatorname{sen}^2 t\,dt} = \boxed{\text{p.p. FD } -\operatorname{sen} t\,dt}$$

$$= 25 \left[\operatorname{sen} t \cdot \cos t - \int \cos t \cdot \cos t\,dt \right]$$

$$= 25 \left[\operatorname{sen} t \cos t - \int (1 - \operatorname{sen}^2 t)\,dt \right] =$$

$$= 25 \left[\operatorname{sen} t \cos t - t + \boxed{\int \operatorname{sen}^2 t\,dt} \right] =$$

$$\Rightarrow \quad 25 \int -\operatorname{sen}^2 t\,dt = 25 \cdot \frac{\operatorname{sen} t \cos t - t}{2} + c =$$

$$= \frac{5 \operatorname{sen} t \, 5 \cos t - 25t}{2} + c$$

sostituisco $t = \arccos \frac{x}{5}$

$$\begin{cases} x = 5\cos t \\ y = 5 \operatorname{sen} t \end{cases} \quad 0 \le t \le \pi$$

$$\int \sqrt{25-x^2}\,dx = \frac{1}{2} \left(x\sqrt{25-x^2} - 25 \arccos \frac{x}{5} \right) + c$$

$$\int \frac{\cos x - x \operatorname{sen} x}{x^2 - (x \operatorname{sen} x)^2}\,dx =$$

$$x^2 - (x \operatorname{sen} x)^2 = x^2 - x^2 \operatorname{sen}^2 x = x^2(1 - \operatorname{sen}^2 x) =$$

$$= x^2 \cos^2 x$$

$$(x \cos x)' = \cos x + x(-\operatorname{sen} x) = \cos x - x \operatorname{sen} x$$

$$= \int \frac{(x\cos x)'}{x^2\cos^2 x}\,dx = \qquad \begin{array}{l} x\cos x = t \\ (x\cos x)'\,dx = dt \end{array}$$

$$= \int \frac{dt}{t^2} = -\frac{1}{t} + c = -\frac{1}{x\cos x} + c$$

$$\int \frac{e^x \ln(2+e^x)}{(e^x+1)^2}\,dx = \boxed{\begin{array}{l} e^x = t \\ e^x dx = dt \end{array}} =$$

$$= \int \frac{\ln(2+t)}{(t+1)^2}\,dt = \boxed{\begin{array}{l} t+1 = s \\ dt = ds \end{array}} = \int \frac{\ln(1+s)}{s^2}\,ds$$

per parti

$$= -\frac{1}{s}\ln(1+s) - \int -\frac{1}{s} \cdot \frac{1}{1+s}\,ds =$$

$$= -\frac{1}{s}\ln(1+s) + \int \frac{1}{s(1+s)}\,ds =$$

$$= -\frac{1}{s}\ln(1+s) + \int \left(\frac{1}{s} - \frac{1}{s+1} \right) ds =$$

$$= -\frac{1}{S}\ln(1+s) + \ln|S| - \ln|S+1| + c =$$

$$= -\frac{1}{s}\ln(1+s) - \ln(1+s) + \ln|S| + C =$$

$$\boxed{s = e^x + 1}$$

$$= \left(1 - \frac{1}{e^x+1}\right)\left(\ln(2+e^x)\right) + \ln(1+e^x) + C$$

$$\int x \operatorname{sen}(x^2)dx = \frac{1}{2}\int 2x \operatorname{sen}(x^2)\,dx =$$

$$= -\frac{1}{2}\cos(x^2) + C$$

con la sostituzione $t = x^2 \quad dt = 2x\,dx$

$$\int e^x (\operatorname{sen} x)^2 dx =$$

$\cos 2x = 1 - 2\operatorname{sen}^2 x$

$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$

$$= \frac{1}{2}\int e^x (1-\cos 2x)\,dx =$$

$$= \frac{1}{2}e^x - \frac{1}{2}\int e^x \cos 2x\,dx = \text{pp.}$$

$$= \frac{1}{2}e^x - \frac{1}{2}\left(e^x \cos 2x - \int e^x(-2\operatorname{sen} 2x)\,dx\right)$$

$$= \frac{1}{2}e^x - \frac{1}{2}\left(e^x \cos 2x + 2e^x \operatorname{sen} 2x - 2\int e^x \cdot 2\cos 2x\,dx\right)$$

$$= \frac{1}{2}e^x - \frac{1}{2}\left(e^x \cos 2x + 2e^x \operatorname{sen} 2x - 4\int e^x \cos 2x\,dx\right)$$

$$5\int e^x \cos 2x\,dx = C + (e^x\cos 2x + 2e^x \operatorname{sen} 2x)$$

$$\int e^x \cos 2x\,dx = \frac{e^x\cos 2x + 2e^x \operatorname{sen} 2x}{5} + c$$

e quindi

$$\frac{1}{2}\int e^x(1-\cos 2x)\,dx = \frac{1}{2}e^x - \frac{e^x\cos 2x + 2e^x \operatorname{sen} 2x}{10} + C$$

Quando devo calcolare un integrale per scomposizione o i due integrali da sommare sono di semplice calcolo oppure conviene fare i conti su ciascun addendo.

$f(x) = x\sqrt{x^2-1}$ definita per $x \in (-\infty,-1] \cup [1,+\infty)$

limiti: $f(x)$ è continua da destra in -1 / sinistra in 1 $\Rightarrow$ i limiti coincidono con i valori della funzione

$f(1) = f(-1) = 0$. Segno $f(x) > 0$ per $x > 1$, < 0 per $x < -1$

$\lim_{x\to+\infty} x\sqrt{x^2-1} = +\infty$ circa come $x \cdot x = x^2 \Rightarrow$ non ci sono asintoti

$\lim_{x\to-\infty} x\sqrt{x^2-1} = -\infty$ " $x|x| = -x^2 \Rightarrow$ non ci sono asintoti

Monotonia?

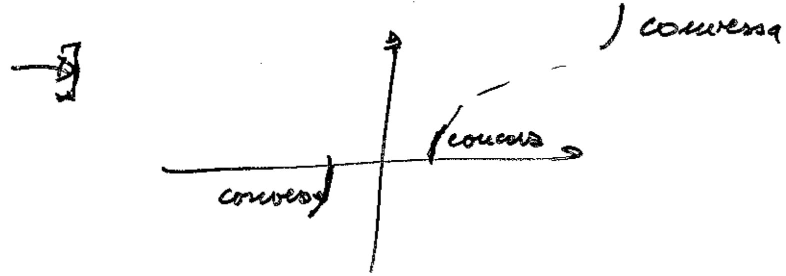
$$f'(x) = \sqrt{x^2-1} + x \cdot \frac{2x}{2\sqrt{x^2-1}} = \frac{x^2-1+x^2}{\sqrt{x^2-1}} =$$

$$= \frac{2x^2-1}{\sqrt{x^2-1}} > 0 \quad \forall x \in \text{I.D.}$$ poiché $x^2 > 1$ perché sia definita f

eq. della retta tangente in $(-1,0)$ e in $(1,0)$

$$\lim_{x\to -1^-} f'(x) = \lim_{x\to -1^-} \frac{1}{\sqrt{(1-x)(-1-x)}} = \lim_{x\to -1^-} \frac{1}{\sqrt{2}\,0^+} = +\infty$$

$$\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} \frac{1}{\sqrt{(x-1)(x+1)}} = \lim_{x\to 1^+} \frac{1}{\sqrt{2}\,0^+} = +\infty$$



concava

flessi?

$$f'(x) = \frac{2x^2-1}{\sqrt{x^2-1}}$$

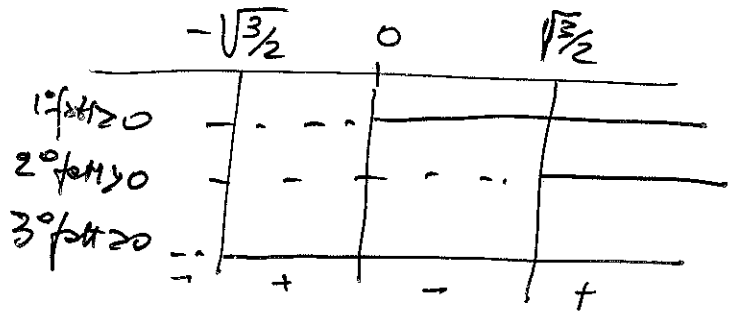
$$f''(x) = \frac{4x\sqrt{x^2-1} - (2x^2-1)\cdot\frac{x}{\sqrt{x^2-1}}}{x^2-1} =$$

$$= \frac{4x(x^2-1) - x(2x^2-1)}{(x^2-1)^{3/2}} =$$

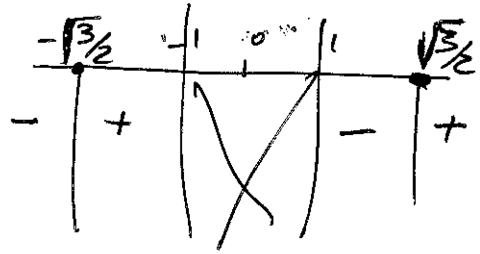
$$= \frac{x(2x^2-3)}{(x^2-1)^{3/2}} > 0$$

$D > 0$ ove f'' è definita

$N = x(\sqrt{2}x - \sqrt{3})(\sqrt{2}x + \sqrt{3}) > 0$



$\frac{N}{D} > 0$: I.O.



$f(\sqrt{3/2}) = \ldots$

$f(-\sqrt{3/2}) = \ldots$

Calcolo della tang. nei flessi...