(1)

$$\int \frac{1}{4-3\sqrt{x}}\,dx$$

$x \geq 0 \quad 4 \neq 3\sqrt{x}$ cioè $x \neq \frac{16}{9}$

$4-3\sqrt{x} = t$

$4-t = 3\sqrt{x}$ $\quad t \leq 4$

$x = \frac{(4-t)^2}{9} \qquad dx = \frac{-2}{9}(4-t)\,dt$

due possibili intervalli in cui calcolare la primitiva; primo: $[0, 16/9)$, secondo $(16/9, +\infty)$

$$\int \frac{1}{4-3\sqrt{x}}\,dx = \int \frac{-\frac{2}{9}(4-t)\,dt}{t} = -\frac{2}{9}\int\left(\frac{4}{t}-1\right)dt$$

$$= -\frac{2}{9}\left(4\ln|t| - t\right) + c =$$

$$= -\frac{2}{9}\left(4\ln|4-3\sqrt{x}| - 4 + 3\sqrt{x}\right) + c.$$

$$= -\frac{8}{9}\ln|4-3\sqrt{x}| - \frac{2}{3}\sqrt{x} + k$$

$$\left(k = \frac{8}{9} + c\right)$$

le prim. in $[0, \frac{16}{9})$ hanno la forma

$$-\frac{8}{9}\ln(4-3\sqrt{x}) - \frac{2}{3}\sqrt{x} + k$$

le prim. in $(\frac{16}{9}, +\infty)$ hanno la forma

$$-\frac{8}{9}\ln(3\sqrt{x}-4) - \frac{2}{3}\sqrt{x} + k.$$

(2)

$$f(x) = \begin{cases} \dfrac{x(e^{1/x}-1)}{1-e^{-2x}} & \text{se } x<0 \\[2ex] \dfrac{3x^2+\sin\frac{x}{2}}{hx} & \text{se } x>0 \end{cases}$$

per quali valori di h è possibile definire $f(0)$ in modo che f sia continua in $x_0 = 0$?

per quali k $\exists\ell$ finito t.c. $\displaystyle\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \ell$?

per tali k basta prendere $f(0)=\ell$ e si ha una funz. continua in $x_0=0$.

$$\lim_{x\to 0^-} \frac{x(e^{1/x}-1)}{1-e^{-2x}} = \lim_{x\to 0^-} \frac{-1\cdot x}{1-e^{-2x}} =$$

$$= \lim_{x\to 0^-} \frac{x}{e^{-2x}-1} = \lim_{x\to 0^-} \frac{x}{-2x} = \boxed{-\frac{1}{2}}$$

$\boxed{e^{-2x}-1 \sim -2x \text{ se } x\to 0}$

$$\lim_{x\to 0^+} \frac{3x^2+\sin x/2}{hx} = \lim_{x\to 0^+} \frac{3x^2+\frac{x}{2}+o\left(\frac{x}{2}\right)}{hx} =$$

$\boxed{\text{per } t\to 0 \ \sin t = t + o(t)}$ $\quad t = \frac{x}{2}$

$$= \lim_{x\to 0^+} \frac{\frac{x}{2}+o(x)+3x^2}{hx} = \lim_{x\to 0^+} \frac{1}{2h} + o(1) = \frac{1}{2h}$$

f è continua in $x_0=0$ per $h=-1$

Ricordo che per $x\to 0$ $x^2 = o(x)$ perché $\displaystyle\lim_{x\to 0}\frac{x^2}{x} = 0$

$$\int e^{-x}(x^2+x)\,dx \underset{pp.}{=} \boxed{FF\ x^2+x}$$

$g'f = e^{-x}(x^2+x)$

$g = -e^{-x}$

$f' = 2x+1$

$$= -e^{-x}(x^2+x) - \int -e^{-x}\cdot(2x+1)\,dx \underset{pp}{=}$$

$$= -e^{-x}(x^2+x) - \left(e^{-x}(2x+1) - \int e^{-x}\cdot 2\,dx\right)$$

$$= -e^{-x}(x^2+x) - e^{-x}(2x+1) - 2e^{-x} + c =$$

$$= -e^{-x}(x^2+3x+3) + c$$

FATE LA VERIFICA

$$F(x) = -e^{-x}(x^2+3x+3) + c$$

$$F'(x) = e^{-x}(x^2+3x+3) + (-e^{-x})(2x+3) =$$

$$= e^{-x}(x^2+x)$$

$$h(x) = \begin{cases} a(-x)^b \ln\left(\dfrac{x^2+1}{1-x}\right) & \text{se } x<0 \\[2ex] \dfrac{3x}{x^2-\tan(x^3+2x)} & \text{se } x>0 \end{cases}$$

per quali a, b posso def $h(0)$ in modo che $h(x)$ sia cont in $x_0=0$

$$\lim_{x\to 0^-} h(x) \underset{?}{=} \lim_{x\to 0^+} h(x)$$

$$\lim_{x\to 0^+} \frac{3x}{x^2-\tan(x^3+2x)} =$$

poiché $x^3+2x \to 0$

$$= \lim_{x\to 0^+} \frac{3x}{x^2-(x^3+2x+o(x^3+2x))} =$$

$$= \lim_{x\to 0^+} \frac{3x}{-2x+x^2-x^3+o(x)} = \lim_{x\to 0^+}\frac{3x}{-2x} =$$

$$= -3/2$$

$$\lim_{x\to 0^-} a(-x)^b \ln\left(\frac{x^2+1}{1-x}\right) = -\frac{3}{2} \quad ?$$

$$\boxed{\frac{x^2+1}{1-x} = 1 + t} \qquad \frac{x^2+1}{1-x}\to 1, \quad t \to 0$$

$$t = \frac{x^2+\cancel{1}-\cancel{1}+x}{1-x}. \text{ Per } x\to 0:$$

$$\ln\left(1+\frac{x^2+x}{1-x}\right) \sim \frac{x^2+x}{1-x} = x\left(\frac{1+x}{1-x}\right)$$

$$\lim_{x\to 0^-} h(x) = \lim_{x\to 0^-} a(-x)^b\cdot x\cdot\frac{1+x}{1-x} =$$

$$\frac{1+x}{1-x} \to 1$$

$$= \lim_{x\to 0^-} (-a)(-x)^{b+1} = -\frac{3}{2} \quad ?$$

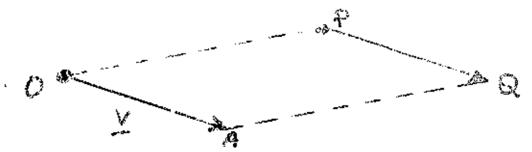
$$(-x)^{b+1} = 1 \iff b+1=0 \iff b=-1$$

$$-a = -\frac{3}{2} \iff a = \frac{3}{2}$$

VETTORI

grandezze individuate da : MODULO (o norma) : reale ≥ 0
DIREZIONE
VERSO.

Rappresentazione: FRECCE USCENTE DA UN PUNTO FISSATO DELLO SPAZIO : O



···· Traslazione $\underline{v}$ da O ad A
o equivalentemente da P a Q $\overrightarrow{OA}=\overrightarrow{PQ}$
o ······

identificazione di vettori EQUIPOLLENTI

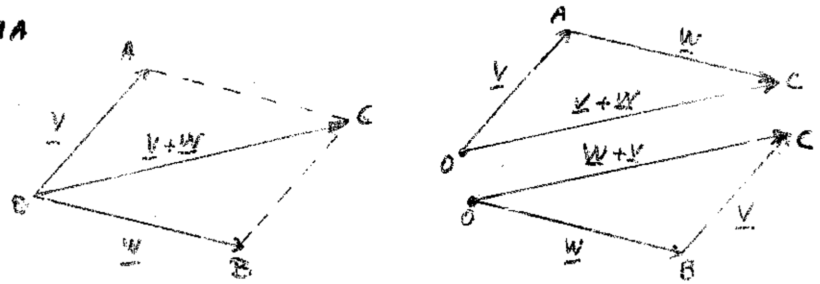
$|\underline{v}|$

se $|\underline{v}| = 0$ dico che $\underline{v}$ è il vettore nullo

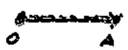
OPERAZIONI

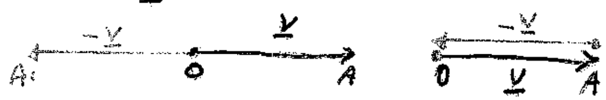
SOMMA

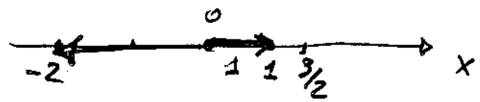
$\underline{v}+\underline{w}$



- commutativa
- associativa
- neutro : vettore nullo : $\underline{O}$
- opposto

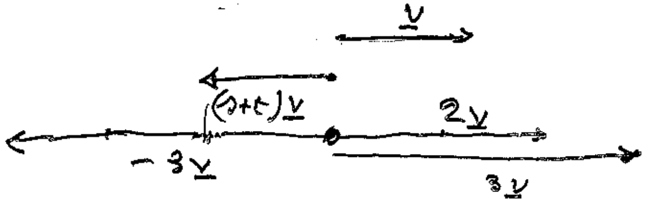




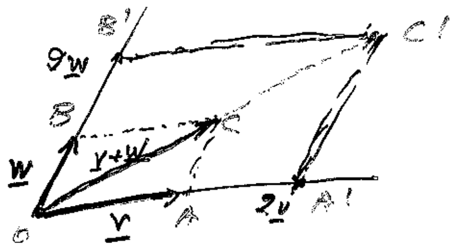


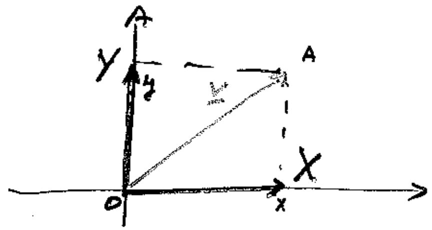
$(s+t)\,\underline{v} = s\underline{v} + t\underline{v}$

$s = 2 \qquad t = -3$



$s\,(\underline{v}+\underline{w}) = s\underline{v} + s\underline{w} \qquad s = 2$





$x \in \mathbb{R}$, $y \in \mathbb{R}$ sono le componenti

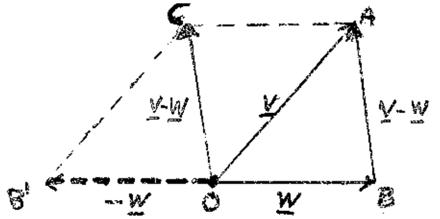
$\overrightarrow{OX} + \overrightarrow{OY} = \overrightarrow{OA}$

$\overrightarrow{OX}$ e $\overrightarrow{OY}$ sono i vettori componenti di $\overrightarrow{OA}$ lungo l'asse x e l'asse y

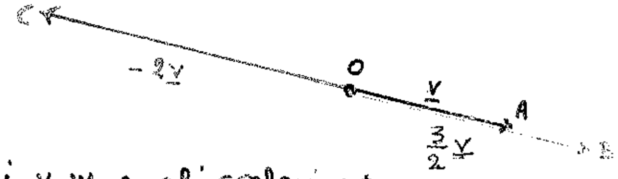
Nella figura precedente $\overrightarrow{OA'}$ è il componente di $\overrightarrow{OC'}$ lungo la direz. di $\underline{v}$ ecc.

DIFFERENZA : $\underline{v} - \underline{w}$

$\vec{OC} = \cancel{\vec{AB}}$ NO $\vec{BA}$



PRODOTTO PER SCALARE $t \in \mathbb{R}$



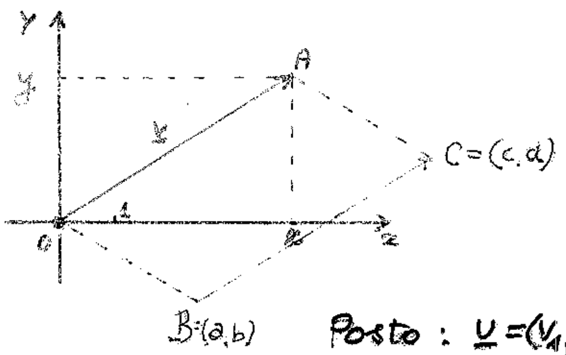
Per tutti i $\underline{v}, \underline{w}$ e gli scalari s, t:

- $(s+t)\underline{v} = s\underline{v} + t\underline{v}$
- $s(\underline{v}+\underline{w}) = s\underline{v} + s\underline{w}$
- $s(t\underline{v}) = (st)\underline{v}$
- $1(\underline{v}) = \underline{v}$

Dirò che i vettori con "SOMMA" e "PRODOTTO PER SCALARE" formano spazio vettoriale su $\mathbb{R}$.

VETTORI COME "n-uple" ORDINATE

Sist. di riferimento nel piano



$\underline{v} = \vec{OA} = (x, y)$

x, y componenti (scalari) di $\underline{v}$

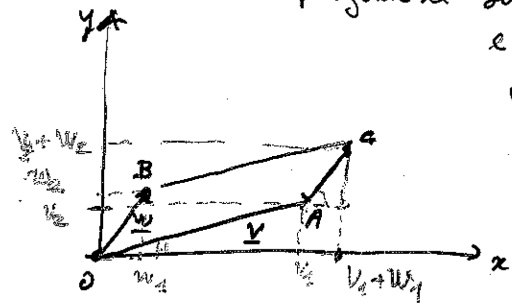
$\vec{OA} = \vec{BC} = (c-a, d-b)$

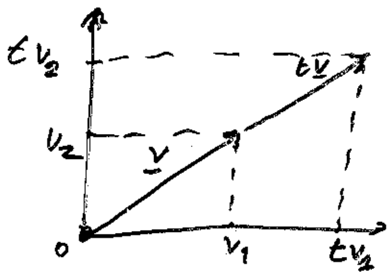
equazioni param. della retta

Posto: $\underline{v} = (v_1, v_2)$, $\underline{w} = (w_1, w_2)$

$$\Rightarrow \underline{v}+\underline{w} = (v_1+w_1, v_2+w_2)$$
$$t\underline{v} = (tv_1, tv_2)$$

Spiegazione su somma di vettori e prodotto scalar-vettore PER COMPONENTI

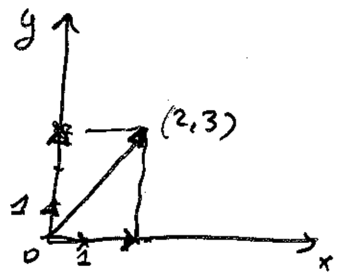




Inoltre:

$$|\underline{v}| = \sqrt{v_1^2 + v_2^2}$$

$$(v_1, v_2) = (2,3) = (2,0) + (0,3)$$
$$= 2(1,0) + 3(0,1)$$



In generale

$$\underline{v} = (v_1, v_2) =$$
$$= v_1(1,0) + v_2(0,1)$$

$(1,0) = \underline{i}$, $(0,1) = \underline{j}$ } versori della base canonica o versori fondamentali

VERSORE $=_{DF}$ VETTORE di MODULO 1

Si ha dunque $\underline{v} = v_1\underline{i} + v_2\underline{j}$ e si dice che ogni vettore $\underline{v}$ è COMBINAZIONE LINEARE di $\underline{i}$ e $\underline{j}$ mediante le proprie componenti.