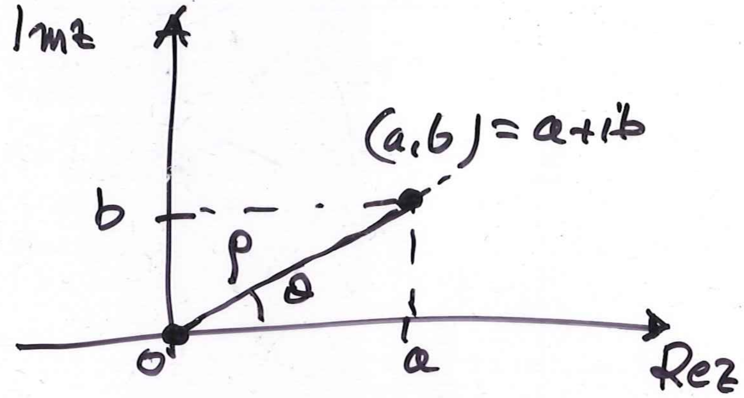
$z = a + ib$




$\forall (a,b) \neq (0,0) \rightarrow (\rho,\theta) \in (0,+\infty) \times (-\pi, \pi]$

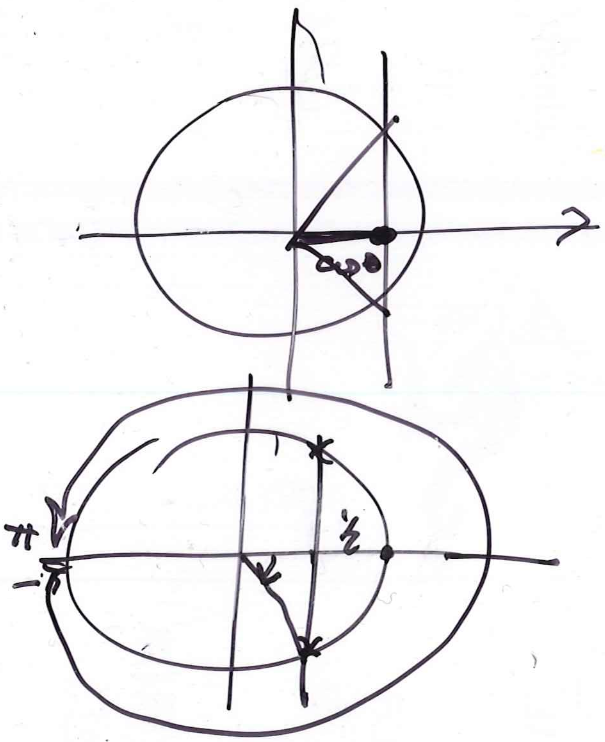
$a = \rho \cos\theta$

$b = \rho \,\mathrm{sen}\,\theta$

$\rho = \sqrt{a^2+b^2}$

$$z = \rho(\cos\theta + i\,\mathrm{sen}\,\theta) = \underbrace{\rho\cos\theta}_{a} + i\,\underbrace{\rho\,\mathrm{sen}\,\theta}_{b}$$

$$\cos\theta = \frac{a}{\rho} = \frac{a}{\sqrt{a^2+b^2}} \qquad \mathrm{sen}\,\theta = \frac{b}{\rho} = \frac{b}{\sqrt{a^2+b^2}}$$




$z = 1 - \sqrt{3}\, i$

$\rho = \sqrt{1+(\sqrt{3})^2} = \sqrt{4} = 2$

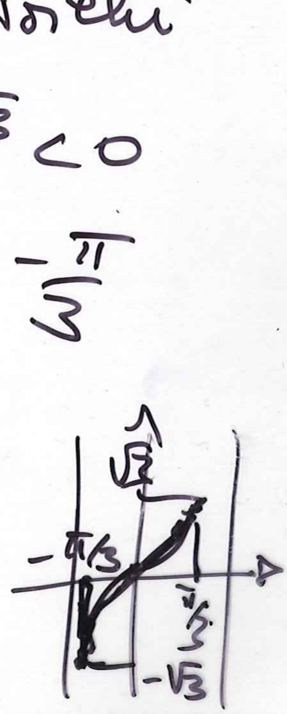
$\cos\theta = \frac{1}{2}$ . Poichè

$\mathrm{sen}\,\theta = -\frac{\sqrt{3}}{2} < 0$

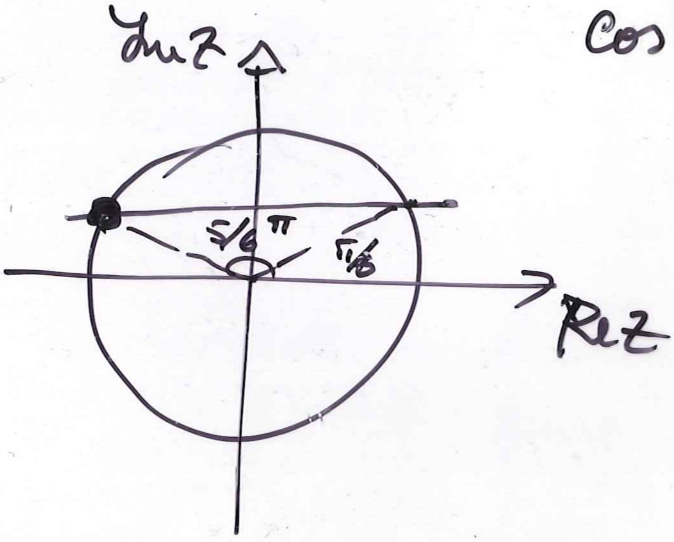
si ha $\theta = -\frac{\pi}{3}$

$\mathrm{tg}\,\theta = -\sqrt{3}$

$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$



$z = -\sqrt{3} + i$ $\qquad |z| = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$



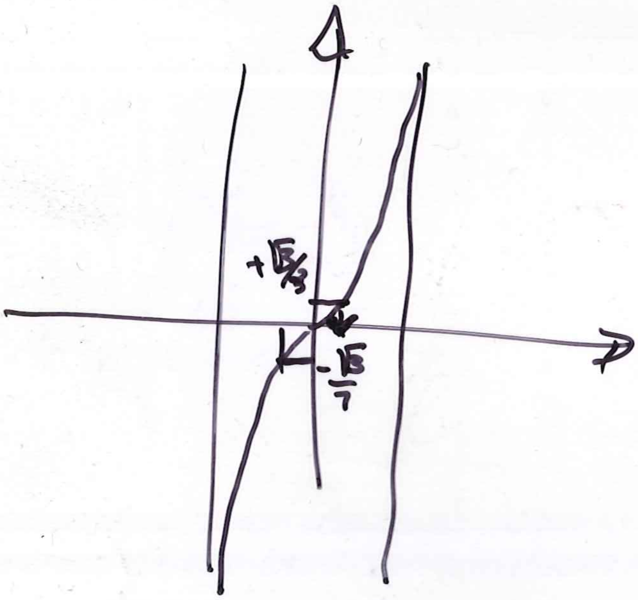

$\cos\theta = \frac{-\sqrt{3}}{2} \qquad \operatorname{sen}\theta = \frac{1}{2}$

$\theta = \frac{5}{6}\pi$

invece

$\operatorname{tg}\theta = \frac{1/2}{-\sqrt{3}/2} = -\frac{\sqrt{3}}{3}$

$\theta = \frac{-\pi}{6}$




arctg: $\mathbb{R} \longrightarrow \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

ma $(-\sqrt{3}, 1)$ sta nel 2° quadrante e quindi il suo argom. principale è compreso in $\left(\frac{\pi}{2}, \pi\right]$

quindi devo traslare di $\pi$ $\Rightarrow \frac{-\pi}{6} + \pi = \frac{5}{6}\pi$

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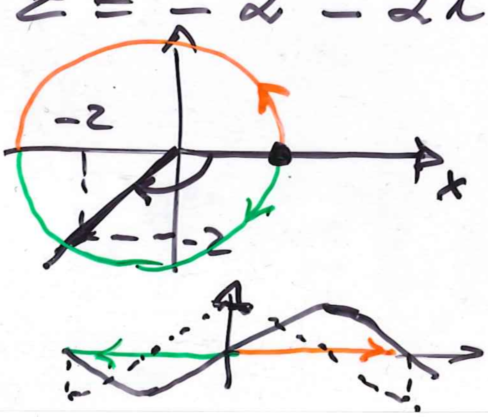
$z = -2 - 2i$ $\qquad |z| = \sqrt{4+4} = 2\sqrt{2}$




$\arg z = \frac{-3\pi}{4}$ princ.

$\cos\theta = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$ $\qquad \frac{3\pi}{4}$

$\operatorname{sen}\theta = \frac{-1}{\sqrt{2}} < 0$ $\qquad -\frac{3\pi}{4}$

Scrivere in forma algebrica

$$z = -2-3i \qquad w = -3+4i \quad : \quad \frac{z}{w} = ?$$

$$\bar{w} = -3-4i$$

$$\frac{z}{w} = \frac{z\cdot\bar{w}}{w\cdot\bar{w}} = \frac{z\cdot\bar{w}}{|w|^2} = \frac{(-2-3i)(-3-4i)}{9+16} =$$

$(2+3i)(3+4i)$

$$= \frac{6-12+17i}{25} = -\frac{6}{25}+\frac{17i}{25}$$

$$z = \frac{1\cdot(-i)}{(-i\cdot i^9)(3+2i)^2} = \frac{-i}{(3+2i)^2} = \frac{-i(3-2i)^2}{[(3+2i)(3-2i)]^2} =$$

$$= \frac{-i(9-4-12i)}{169} = -\frac{5i}{169} - \frac{12}{169}$$

$$z = \sqrt{3}+i \implies z^{13} =$$

$$|z| = \sqrt{3+1} = 2$$

$$\arg z = \frac{\pi}{6} + 2k\pi \qquad k\in\mathbb{Z}$$

$$= 2^{13}\left(\cos\frac{\pi\cdot 13}{6} + i\sin\frac{13\pi}{6}\right) =$$

$$\boxed{\frac{13}{6}\pi = \frac{\pi}{6} + 2\pi}$$

$$= 2^{13}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) =$$

$$= 2^{12}\sqrt{3} + 2^{12}i$$

ES. 14.1 → 14.5.

$\operatorname{sen} x + \cos x > 0$

2 strade ... anzi 3!

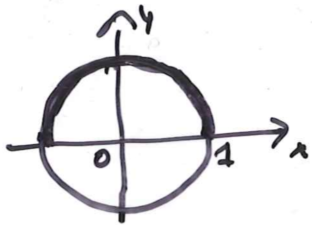
1) strada sicura. La diseq. equivale a

$$\frac{\sqrt{2}}{2} \operatorname{sen} x + \frac{\sqrt{2}}{2} \cos x > 0$$

uso le formule di addizione ricordando che $\sqrt{2}/2 = \cos\frac{\pi}{4} = \operatorname{sen}\pi/4$

$$\Updownarrow$$

$$\operatorname{sen}\left(x + \frac{\pi}{4}\right) > 0$$




$$\Updownarrow$$

$$0 + 2k\pi < x + \pi/4 < \pi + 2k\pi$$

Soluzione: $-\pi/4 < x < 3\pi/4 + 2k\pi$

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2) Strada spontanea e ... artistica.

$\operatorname{sen} x > -\cos x$

VORREI DIVIDERE per $\cos x$ e riportarmi alla tangente
DEVO dividere in casi:

A) $\cos x = 0 \Rightarrow \operatorname{sen} x = \pm 1$. Ovviamente $-1 \not> 0$
quindi la sola soluz è $\operatorname{sen} x = 1$
$\Rightarrow x = \pi/2 + 2k\pi$

B) $\cos x \neq 0$. Ho due sistemi, a seconda del segno:

$$\begin{cases} \cos x > 0 \\ \operatorname{tg} x > -1 \end{cases} \quad \text{oppure} \quad \begin{cases} \cos x < 0 \\ \operatorname{tg} x < -1 \end{cases}$$

Ricordo che $\operatorname{tg} x$ è funzione crescente e quindi conserva il verso della disug.
Allora in $[-\pi/2, 3\pi/2]$ i due sistemi equivalgono a

$$\begin{cases} x \in (-\pi/2, \pi/2) \\ x \in (-\pi/4, \pi/2) \cup (3\pi/4, 3\pi/2) \end{cases} \quad \text{opp.} \quad \begin{cases} x \in (\pi/2, 3\pi/2) \\ x \in (-\pi/2, -\pi/4) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \end{cases}$$

cioè

$$x \in (-\pi/4, \pi/2) \quad \text{oppure} \quad x \in (\pi/2, 3\pi/4)$$

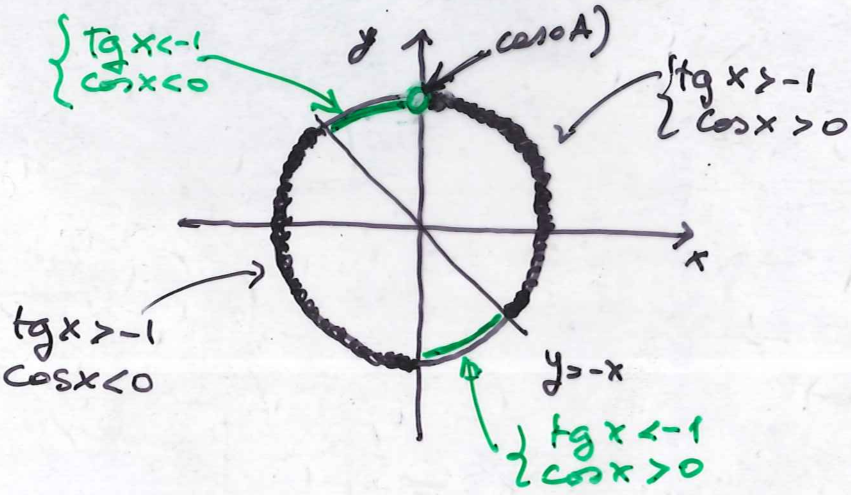
Ora ricordo la periodicità:

sull'asse reale le soluzioni nel caso B) sono

$$x \in \left(-\frac{\pi}{4} + 2k\pi, \frac{\pi}{2} + 2k\pi\right) \cup \left(\frac{\pi}{2} + 2k\pi, \frac{3\pi}{4} + 2k\pi\right)$$

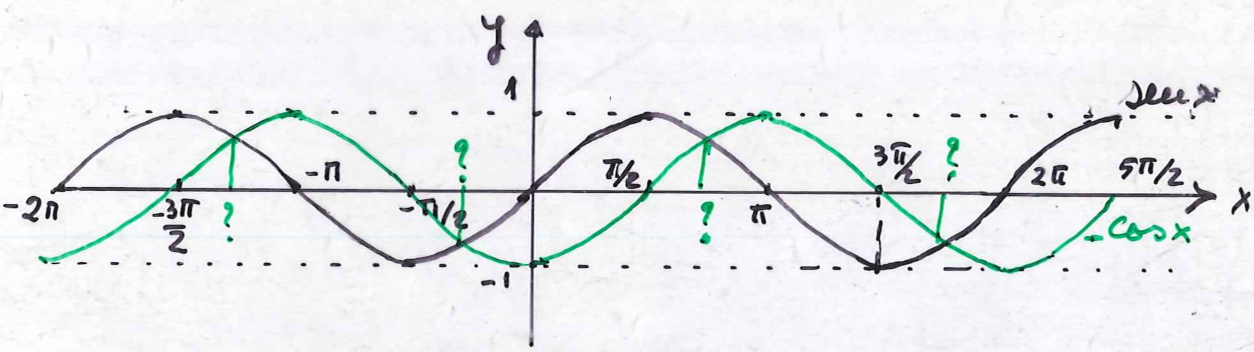
Infine unisco la situazione del caso A) con quella del caso B) e trovo:

$$x \in \left(-\pi/4 + 2k\pi, 3\pi/4 + 2k\pi\right)$$




3) Strada grafica.

sen x > - cos x . Posso confrontare i due grafici

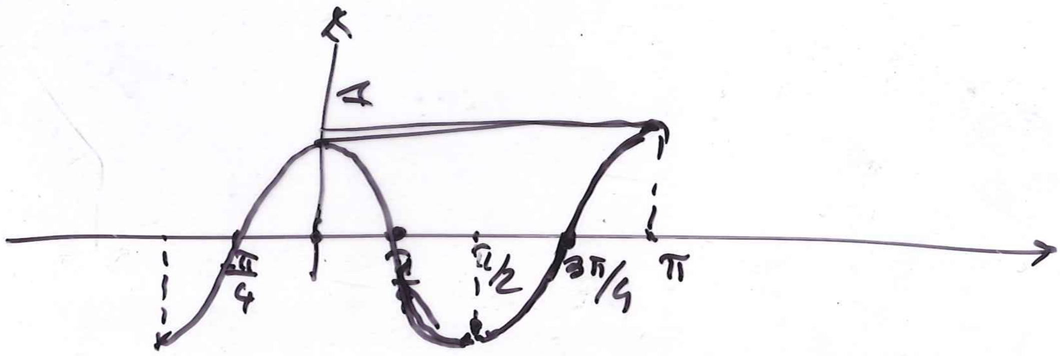



E' chiaro dal grafico (???) che il grafico di sen x sta "sopra" quello di -cos x in certi intervalli ... si tratta di individuare dove sono le intersezioni che, per motivi di simmetria si troveranno nel punto medio dell'intervallo $\left(-\frac{\pi}{2}, 0\right)$, in quello dell'intervallo $(\pi/2, \pi)$ ecc.

Ovviamente vengono fuori le stesse soluzioni!

$\cos 2x$

$$x \xrightarrow{(\cdot)2} 2x \xrightarrow{\cos} \cos 2x$$

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$$\underset{\text{mai}<0}{\sqrt{x^2-x}} > 2x$$

I.D. $x^2-x \geq 0$ cioè $(-\infty,0]\cup[1,+\infty)$

$\Rightarrow$ in $(-\infty,0)$ $\sqrt{x^2-x} > 0 > 2x$ perché $x<0$

in $x=0$ $\sqrt{x^2-x}$ vale $0$ e non è $> 2\cdot 0 = 0$

$\sqrt{x^2-x} \geq 0$ $\quad 2x \geq 1 > 0$

Quindi posso quadrare $\Rightarrow$ $0<a<b \Leftrightarrow a^2<b^2$

$$\begin{cases} x^2-x > 4x^2 \\ x \geq 1 \end{cases} \quad \begin{cases} 3x^2+x < 0 \\ x \geq 1 \end{cases} \quad \begin{cases} x \in (-\frac{1}{3},0) \\ x \geq 1 \end{cases}$$

MAI

$\Rightarrow$ SOL DISEQ: $(-\infty,0)$

$\sqrt{x^2+4x} < x$

I.D. $(-\infty,-4] \cup [0,+\infty)$

MAI VERO

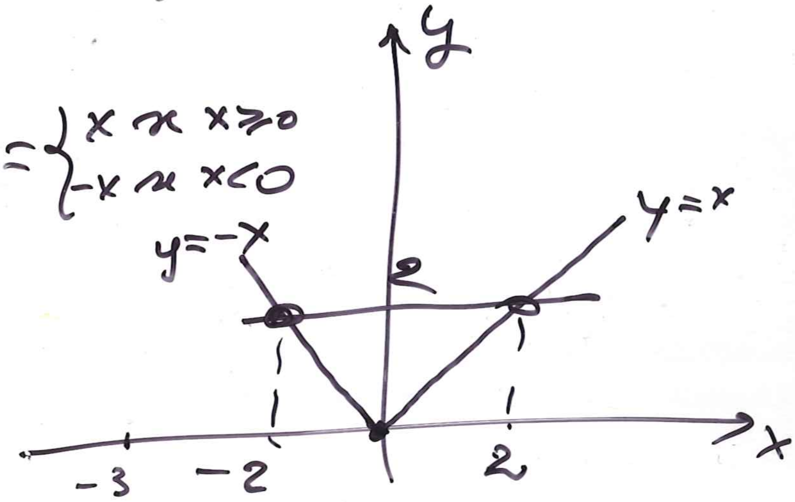
Devo considerare solo $x \in [0,+\infty)$

$x^2+4x < x^2 \iff 4x<0$ mai

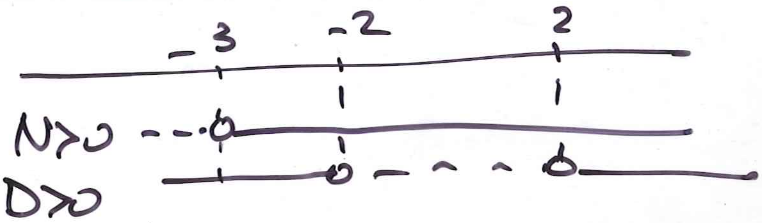
---

I.D. $\log_2\left(\dfrac{x+3}{|x|-2}\right)$

Grafico di $|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x<0 \end{cases}$




$$\frac{x+3}{|x|-2} > 0$$




Sol: $(-3,-2)\cup(2,+\infty)$

$$\begin{cases} \dfrac{2^{2x}-1}{x-5} \le 0 \\ \sqrt{x^2-2x-3}+2>0 \end{cases}$$

$$2^{2x}-1>0 \iff 2^{2x}>1 \overset{\log_2}{\iff} 2x>0 \iff x>0$$

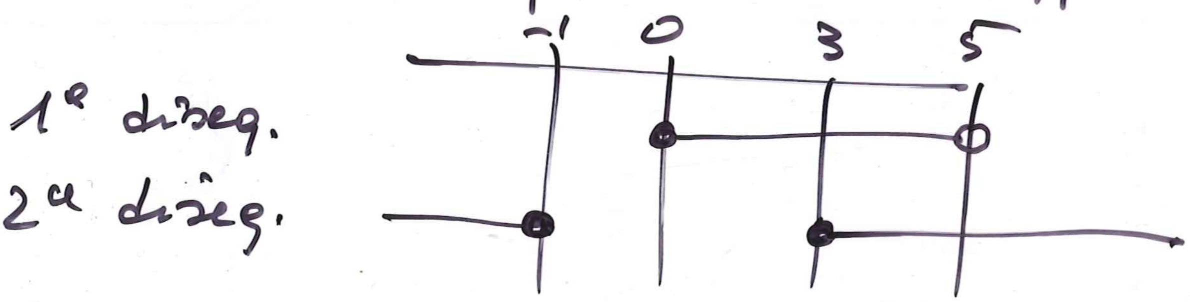
$x-5>0$ se $x>5$


$1^a$ disq è soddisfatta in $[0,5)$

$\sqrt{x^2-2x-3}+2>0$ sempre purché sia definita la radice

cioè per $x^2-2x-3 \geq 0$

cioè per $x \le -1$ oppure $x \geq 3$




Quindi il sistema ha soluz. per $x \in [3,5)$

Fare 2.10