ES. 11.23 Mostrare che

$\forall h \in \mathbb{R}$ si ha

$$\underline{u} \wedge (\underline{v} + h\underline{u}) = \underline{u} \wedge \underline{v} = (\underline{u} + h\underline{v}) \wedge \underline{v}$$

$$(\underline{u} \wedge (\underline{v} + h\underline{u})) \cdot \underline{w} = (\underline{u} \wedge \underline{v}) \cdot \underline{w} = (\underline{u} \wedge \underline{v}) \cdot (\underline{w} + h\underline{u})$$

1ª)

$$\underline{u} \wedge (\underline{v} + h\underline{u}) \overset{\text{DISTR.}}{=} \underline{u} \wedge \underline{v} + \underline{u} \wedge (h\underline{u}) \overset{\text{OMOG}}{=}$$

$$= \underline{u} \wedge \underline{v} + h(\underline{u} \wedge \underline{u}) = \underline{u} \wedge \underline{v} + \underline{0} = \underline{u} \wedge \underline{v}$$

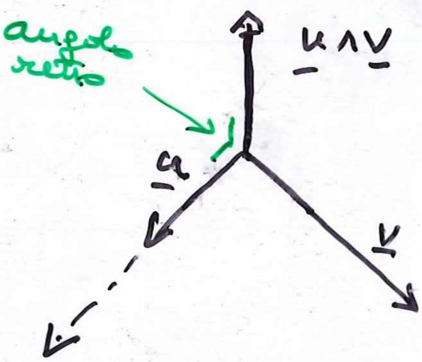
$$(\underline{u} + h\underline{v}) \wedge \underline{v} = \underline{u} \wedge \underline{v} + h(\underline{v} \wedge \underline{v}) = \underline{u} \wedge \underline{v}$$

$\Longrightarrow$ Multilinearità del determinante.

2ª

$$(\underline{u} \wedge (\underline{v} + h\underline{u})) \cdot \underline{w} = (\underline{u} \wedge \underline{v}) \cdot \underline{w}$$

$$(\underline{u} \wedge \underline{v}) \cdot (\underline{w} + h\underline{u}) = (\underline{u} \wedge \underline{v}) \cdot \underline{w} + \underbrace{(\underline{u} \wedge \underline{v}) \cdot h\underline{u}}_{= 0}$$



$= 0$ perché $\widehat{\underline{u} \wedge \underline{v} \;\; \underline{u}} = \frac{\pi}{2}$

ES. 11. 28.

$$\begin{cases} 2x - y + 3z = 1 \\ y - 4z = 2 \end{cases}$$ è una retta?

$$\begin{cases} 2x - (2+4t) + 3t = 1 \\ y = 2 + 4t \\ z = t \end{cases} \Rightarrow \begin{cases} 2x = 3 + 5t \\ y = 2 + 4t \\ z = t \end{cases}$$

$$\Rightarrow \begin{cases} x = 3/2 + 5/2\, t \\ y = 2 + 4t \\ z = t \end{cases} \qquad P = \left(\tfrac{3}{2}, 2, 0\right) + t\left(\tfrac{5}{2}, 4, 1\right)$$

Sì!

$\underline{v}_r = \left(\frac{5}{2}, 4, 1\right)$, o un vettore proporzionale, dà la direz. della retta

$\underline{v}_x = (1, 0, 0)$ $\qquad \cos \widehat{rx} = \dfrac{\underline{v}_r \cdot \underline{v}_x}{|\underline{v}_r|\,|\underline{v}_x|} = \dfrac{5/2 + 0 + 0}{\sqrt{\frac{25}{4} + 16 + 1} \cdot 1}$

$$= \frac{5/2}{\sqrt{\frac{25+68}{4}}} = \frac{5}{\sqrt{93}}$$

$$\cos \widehat{ry} = \frac{4}{\sqrt{\frac{25}{4} + 16 + 1}} = \frac{4}{\frac{1}{2}\sqrt{93}} = \frac{8}{\sqrt{93}} \qquad \underline{v}_y = (0, 1, 0)$$

$$\cos \widehat{rz} = \frac{1}{\frac{\sqrt{93}}{2} \cdot 1} = \frac{2}{\sqrt{93}} \qquad \underline{v}_z = (0, 0, 1)$$

$$\left(\frac{5}{\sqrt{93}}, \frac{8}{\sqrt{93}}, \frac{2}{\sqrt{93}}\right) = k\,\underline{v}_r \qquad k = \frac{2}{\sqrt{93}}$$

$$|k\underline{v}_3| = \sqrt{\frac{25 + 64 + 4}{93}} = 1$$

ES. 11.34

Eq. della retta per $A=(1,0,0)$, $\perp$ retta r:
$2-x=3y=z$ e // piano π: $x+y+z=0$,

$$\begin{cases} x = 1 + at \\ y = 0 + bt \\ z = 0 + ct \end{cases}$$ ove $(a,b,c) \perp \underline{v}_r = (-1, \frac{1}{3}, 1)$
$\perp \underline{v}_\pi = (1,1,1)$

$$2-x=3y=z=k \Rightarrow \begin{cases} 2-x=k \\ 3y=k \\ z=k \end{cases} \Rightarrow \begin{cases} x=2-k \\ y=\frac{1}{3}k \\ z=k \end{cases}$$

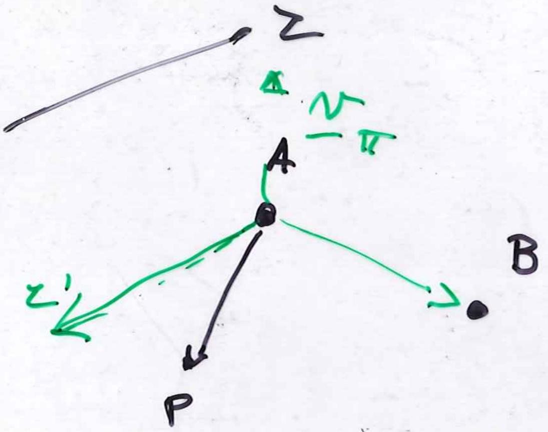
$$(a,b,c) = \underline{v}_r \wedge \underline{v}_\pi = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 1/3 & 1 \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= \underline{i} \begin{vmatrix} 1/3 & 1 \\ 1 & 1 \end{vmatrix} - \underline{j} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} + \underline{k} \begin{vmatrix} -1 & 1/3 \\ 1 & 1 \end{vmatrix} =$$

$$= -\frac{2}{3}\underline{i} + 2\underline{j} - \frac{4}{3}\underline{k} = \left(-\frac{2}{3}, 2, -\frac{4}{3}\right)$$

$$\Rightarrow \begin{cases} x = 1-2t \\ y = 6t \\ z = -4t \end{cases}$$ sono le eq. cercate

Es. 11.39 Eq. del piano per $A=(1,0,0)$, $B=(0,1,0)$ e $//\ r$: $x=y=2z-1$. Dist. da $(0,0,0)$?



$r' // r$, r' passa per A.
cerco un vettore $\perp$ al $\underline{v}_r$ e a $\vec{AB}$.

$\underline{v}_r = (1, 1, \frac{1}{2})$

$\vec{AB} = (0-1, 1-0, 0) =$
$= (-1, 1, 0)$

$$\vec{AP} \cdot (\vec{AB} \wedge \underline{v}_r) = 0$$

$$\begin{vmatrix} x-1 & y-0 & z-0 \\ -1 & 1 & 0 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0$$

Cioè

$$(x-1)\begin{vmatrix} 1 & 0 \\ 1 & \frac{1}{2} \end{vmatrix} - (y-0)\begin{vmatrix} -1 & 0 \\ 1 & \frac{1}{2} \end{vmatrix} + (z-0)\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}(x-1) + \frac{1}{2}y - 2z = 0$$

Controllo: passa per A e B (sostituisco ....)
è $//\ r$ poiché

$$\left(\frac{1}{2}, \frac{1}{2}, -2\right) \cdot \left(1, 1, \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} - 1 = 0$$

Dist. da $(0,0,0)$: $\dfrac{\left|-\frac{1}{2}\right|}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (2)^2}} = \dfrac{1/2}{\sqrt{9/2}} = \dfrac{1}{3\sqrt{2}}$

ES. 11.40 Piano per $A=(0,1,0)$, $//$

$r: x-1 = 2y = -z$ e $\perp \pi : x-y-z=0$

il vettore direzionale del piano cercato deve essere $\perp \underline{v}_r$ e $\perp \underline{v}_\pi$

$\underline{v}_r = (1, \frac{1}{2}, -1)$ $\underline{v}_\pi = (1,-1,-1)$

$$\underline{v}_r \wedge \underline{v}_\pi = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1/2 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

eq. piano : $\vec{AP} \circ (\underline{v}_r \wedge \underline{v}_\pi) = 0$

$$\begin{vmatrix} x-0 & y-1 & z-0 \\ 1 & 1/2 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

$$x \begin{vmatrix} 1/2 & -1 \\ -1 & -1 \end{vmatrix} - (y-1) \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} + z \begin{vmatrix} 1 & 1/2 \\ 1 & -1 \end{vmatrix} = 0$$

$-\frac{3}{2}x + 0\,(y-1) - \frac{3}{2}z = 0$ cioè

$$\boxed{x+z=0}$$

$$\lim_{n\to+\infty} \frac{2n^2-3n}{\sqrt{n^2+n}} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{n\to+\infty} \frac{2n^2\left(1-\frac{3}{2n}\right)}{\sqrt{n^2}\sqrt{1+\frac{1}{n}}} \quad \left(1-\frac{3}{2n}\right)\to 1 \quad \sqrt{1+\frac{1}{n}} \to 1$$

$$\underset{n>0}{=} \lim_{n\to+\infty} \frac{2n^2}{n} = +\infty$$

$$\lim_{n\to+\infty} \left(\frac{n^2+n\ln n}{n^2+1}\right)^n = \left[1^\infty\right]$$

$\left\{\frac{n^2}{n\ln n}\right\} = \left\{\frac{n}{\ln n}\right\} \to +\infty$

n^2 è ∞ di ord. sup. a $(n\ln n)$.

$$= \lim_{n\to+\infty} \left(1+\frac{n\ln n-1}{n^2+1}\right)^n =$$

$$= \lim_{n\to+\infty} e^{n\cdot\ln\left(1+\frac{n\ln n-1}{n^2+1}\right)} =$$

$$\ln\left(1+\frac{n\ln n-1}{n^2+1}\right) \sim \frac{n\ln n-1}{n^2+1}$$

$\Rightarrow$ all'esponente ho succ. asintotica a

$$n\cdot\frac{n\ln n-1}{n^2+1} \sim \frac{\cancel{n^2}\ln n}{\cancel{n^2}} \to +\infty$$

$$= e^{+\infty} = +\infty$$

$$\lim_{n\to+\infty} n \sin\left(\frac{3n}{n^2+1}\right) = [\infty \cdot 0] =$$

$$= \lim_{n\to+\infty} n \cdot \frac{3n}{n^2+1} = 3$$

visto che $\frac{3n}{n^2+1} \to 0$, $\operatorname{sen}\frac{3n}{n^2+1} \sim \frac{3n}{n^2+1}$

e visto che entra in gioco solo 1 prodotto posso sostituire l'asintotico

---

$$\lim_{n\to+\infty} \sqrt{n} \ln\left(1+\frac{2}{n}\right) = [\infty \cdot 0]$$

$\left\{\frac{2}{n}\right\} \to \Rightarrow \ln\left(1+\frac{2}{n}\right) \sim \frac{2}{n}$

e posso sostituire .....

$$= \lim_{n\to+\infty} \sqrt{n} \cdot \frac{2}{n} = 0+$$

$$\lim_{n\to+\infty} n \ln\left(\frac{n+3}{n}\right) =$$

$$= \lim_{n\to+\infty} n \ln\left(1+\frac{3}{n}\right) =$$

$\left\{\frac{3}{n}\right\} \to 0 \ldots$

$$= \lim_{n\to+\infty} n \cdot \frac{3}{n} = 3$$

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{\ln n} = \lim_{n\to+\infty} e^{\ln\left(1+\frac{1}{n}\right)\cdot \ln n}$$

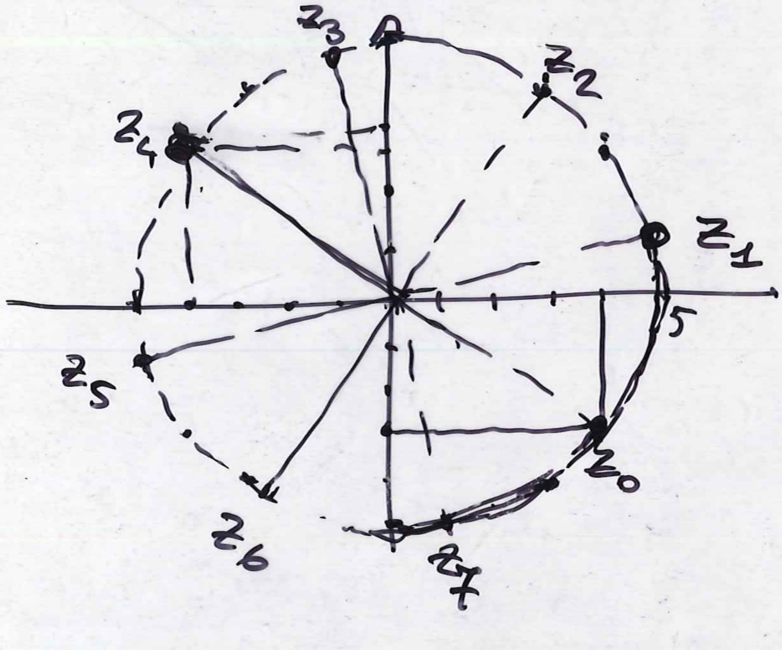
all'esponente $\left\{\frac{1}{n}\right\}\to 0 \Rightarrow \ln\left(1+\frac{1}{n}\right) \sim \frac{1}{n}$

all'esp. ho: $\left\{\frac{\ln n}{n}\right\} \longrightarrow 0$

$$\Rightarrow \lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{\ln n} = e^{0} = 1.$$

ESERCIZIO 14.8

Sia $z_0 = 4-3i$ una radice ottava di un numero complesso α. Altre 7 radici in f.a. dopo averle rappresentate nel piano di A.G.



$$z_1 = z_0\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$
$$= (4-3i)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) =$$
$$= \left(2\sqrt{2} + \frac{3}{2}\sqrt{2}\right) + i\left(2\sqrt{2} - \frac{3}{2}\sqrt{2}\right)$$

$$z_2 = 3+4i$$

$$z_3 = \left(\frac{3}{2}\sqrt{2} - 2\sqrt{2}\right) + i\left(\frac{7}{2}\sqrt{2}\right)$$

$$z_4 = -z_0 = -4+3i$$

$$z_5 = -z_1 = -\frac{7}{2}\sqrt{2} - \frac{\sqrt{2}}{2}i$$

$$z_6 = -3-4i$$

$$z_7 = \frac{\sqrt{2}}{2} - \frac{7}{2}\sqrt{2}\,i.$$

Esercizio 14.5. Trovare modulo, argomento delle radici n-esime e disegnarle sul piano. F.T. e F.A.

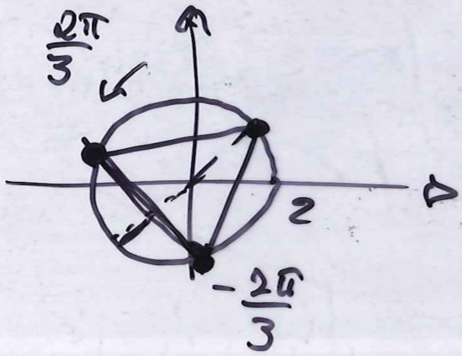
a) $z = 4\sqrt{2}\,i - 4\sqrt{2}$ $\quad \boxed{n=3}$

$|z| = 4\sqrt{2+2} = 8 \Rightarrow \cos\theta = -\frac{\sqrt{2}}{2}, \quad \operatorname{sen}\theta = \frac{\sqrt{2}}{2}$

$\arg z = \frac{3\pi}{4} + 2k\pi$

Radici terze: modulo $\sqrt[3]{8} = 2$

argomento $\frac{\pi}{4} + k\cdot\frac{2\pi}{3} \quad (k = 0, \pm 1)$



$w_0 = 2\left(\cos\frac{\pi}{4} + i \operatorname{sen}\frac{\pi}{4}\right) =$

$= \sqrt{2} + i\sqrt{2}$

$w_1 = w_0 \cdot \left(\cos\frac{2\pi}{3} + i \operatorname{sen}\frac{2\pi}{3}\right) = \dots$

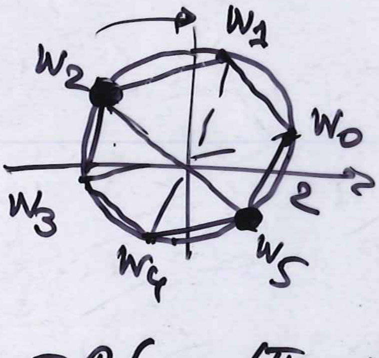
$w_{-1} = w_0 \left(\cos\frac{-2\pi}{3} + i \operatorname{sen}\left(\frac{-2\pi}{3}\right)\right) = \dots$

b) $z = 64 i \quad n = 6$

$|z| = 64 \quad \arg z = \frac{\pi}{2} + 2k\pi$

Radici 6°: modulo $\sqrt[6]{64} = 2$

argom: $\frac{\pi}{12} + k\frac{\pi}{3}$



$k=0 \quad \pi/12$

$k=1 \quad 5\pi/12$

$k=2 \quad 9\pi/12 = 3\pi/4 \quad \dots$

$w_k = 2\left(\cos\left(\frac{\pi}{12} + k\frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi}{12} + k\frac{\pi}{3}\right)\right)$

$k=2 \quad w_2 = 2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right) = -\sqrt{2} + \sqrt{2} i = -w_5$

$-w_4 = w_1 = w_2 \left(\cos\left(\frac{-\pi}{3}\right) + i \operatorname{sen}\left(\frac{-\pi}{3}\right)\right) = \dots$

$-w_0 = w_3 = w_2 \left(\cos\left(\frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi}{3}\right)\right) = \dots$

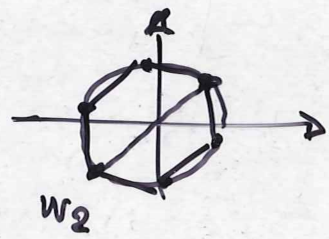
c)

$z = -64i \qquad n = 6$

$|z| = 64 \qquad \arg z = -\frac{\pi}{2} + 2k\pi$

(radici seste : modulo 2

argom. $\frac{-\pi}{12} + k\frac{2\pi}{3}$



$k=0 \quad -\pi/12$

$k=1 \quad 7\pi/12$

$k=2 \quad 15\pi/12 = 5\pi/4$ $\equiv -\frac{3\pi}{4}$ ecc. come sopra.

d) $z = -16 \qquad n = 8$

$|z| = 16 \qquad \arg z = -\pi + 2k\pi$

radici ottave: modulo $\sqrt[8]{16} = \sqrt{2}$

arg. $\frac{-\pi}{8} + k\frac{\pi}{4}$

Serve trovare $\cos\left(\frac{-\pi}{8}\right)$, $\operatorname{sen}\left(\frac{-\pi}{8}\right)$ e poi per rotazioni di $\frac{\pi}{4}$ come nell'es. 14.8.

e) $z = \frac{1}{2} - \frac{\sqrt{3}}{2} i \qquad n = 4$

$|z| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1 \qquad \arg = -\frac{\pi}{3} + 2k\pi$

radici quarte : modulo $\sqrt[4]{1} = 1$

argomento $\frac{-\pi}{12} + k\frac{\pi}{2}$

$-\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{3} \longrightarrow$ calcolare seno e coseno.

ecc.

f) $z = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \qquad n = 3$

$|z| = 1$

$\arg z = -\frac{\pi}{4} + 2k\pi$

radici terze : modulo $\sqrt[3]{1} = 1$

argomento $\frac{-\pi}{12} + \frac{2\pi}{3} \cdot k$

per $k = -1$ ho $-\frac{9}{12}\pi = -\frac{3\pi}{4}$

$$\Rightarrow \quad w_{-1} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$$

$$w_0 = \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right)\left(\cos\frac{2\pi}{3} + i \operatorname{sen}\frac{2\pi}{3}\right)$$

$$w_2 = \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right)\left(\cos\left(\frac{-2\pi}{3}\right) + i \operatorname{sen}\left(\frac{-2\pi}{3}\right)\right)$$

Sv[illegible]re i conti!