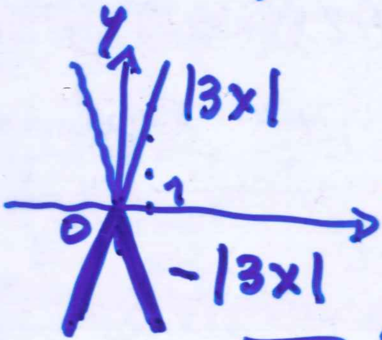
1.23 Quali delle seguenti funz sono INIETTIVE?

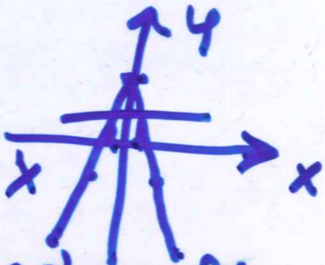
1. D = $\mathbb{R}$

$f(x) = 2-3x$ sì $y = 2-3x \Rightarrow x = \frac{2-y}{3}$

1. D = $\mathbb{R}$

$f(x) = 2 - |3x|$ NO





ad es. $f(x)=0$ per $x = \pm 2/3$

$f(x) = \frac{1}{2-3x}$ sì

1. D. $(-\infty, 2/3) \cup (2/3, +\infty)$ — $y = \frac{1}{2-3x} \Rightarrow 2-3x = \frac{1}{y}$

$\Rightarrow x = \frac{2}{3} - \frac{1}{3y}$

$f(x) = 2x + |2x| =$

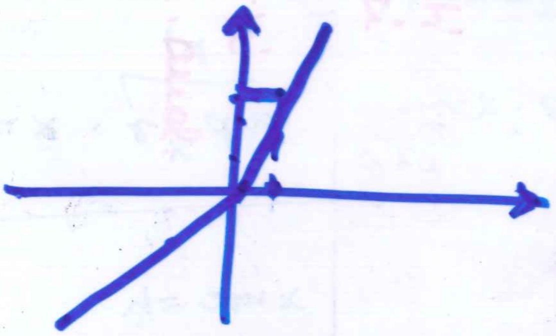
NO $= \begin{cases} 2x+2x & x \geq 0 \\ 2x-2x & x < 0 \end{cases}$

per provare SÌ RISOLVERE rispetto a x l'eq. $y = f(x)$

Se la sol. è unica f è INIETTIVA

per provare NO : trovare due valori x_1, x_2 diversi t.c. $f(x_1) = f(x_2)$

$f(x) = 2x + |x| = \begin{cases} 2x+x & se\ x \geq 0 \\ 2x-x & se\ x<0 \end{cases}$



è iniettiva e biunivoca

1.24

$f(x) = x^2 - 1$ $\qquad \mathbb{R} \xrightarrow{f} [-1,+\infty) = f(\mathbb{R}) \subseteq \mathbb{R}$

$g(x) = \sqrt{x}$ $\qquad [0,+\infty) \xrightarrow{g} [0,+\infty) \subseteq \mathbb{R}$

calcolare f composta g : $g \circ f$

$A \xrightarrow{f} B \xrightarrow{g} C$

$g(f(x)) = g(x^2-1) = \sqrt{x^2-1}$

$I.D.(g \circ f) = (-\infty,-1] \cup [1,+\infty)$

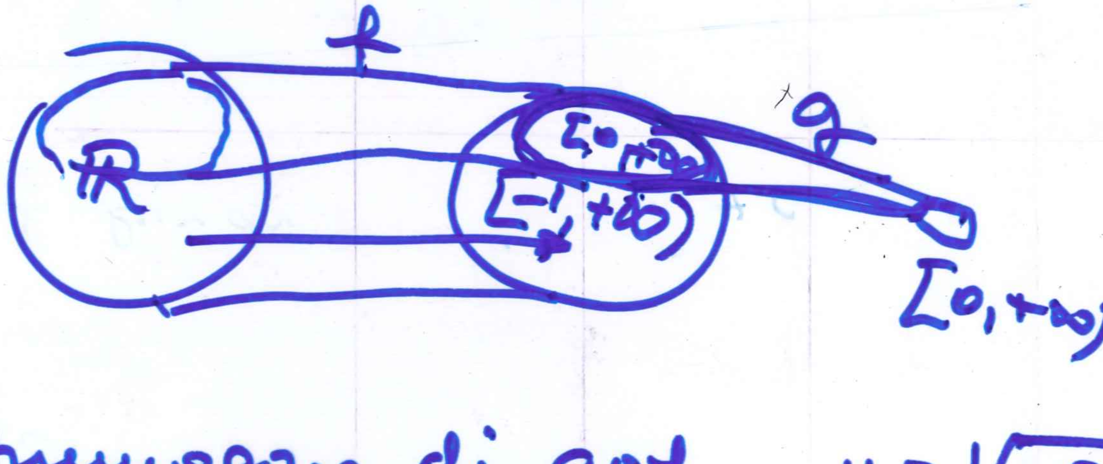


Immagine di $g \circ f$ $\qquad y = \sqrt{x^2-1}$

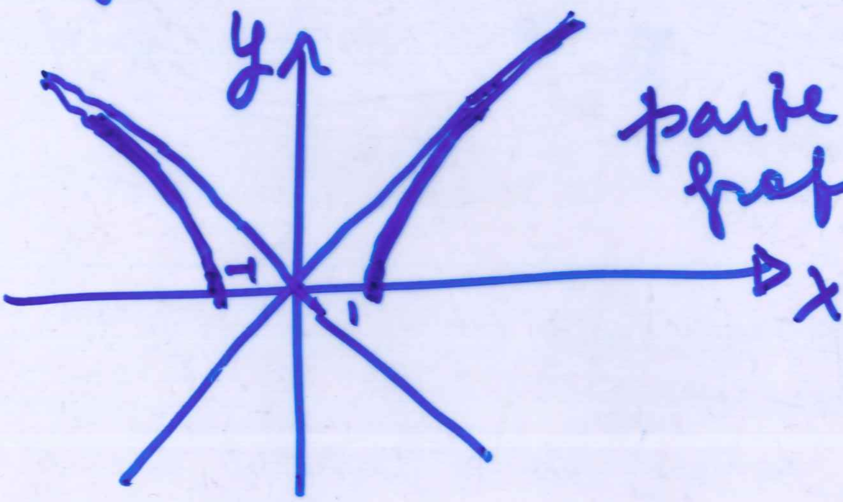
$\begin{cases} y \geq 0 \\ y^2 = x^2 - 1 \end{cases} \Rightarrow \begin{cases} y \geq 0 \\ x^2 = y^2+1 \end{cases}$ $\qquad x = \sqrt{y^2+1}$ $\quad x = -\sqrt{y^2+1}$

sempre! Immagine $[0,+\infty)$

Vedi anche il grafico:

$$y = \sqrt{x^2-1}$$

$$\begin{cases} y^2 = x^2 - 1 \\ y \geq 0 \end{cases} \qquad \begin{cases} \boxed{x^2 - y^2 = 1} \\ y \geq 0 \end{cases}$$



parte superiore del grafico di una iperbole equilatera

1.25 Fatto.

1.26 I.D. $(-\infty, \sqrt[3]{2}) \cup (\sqrt[3]{2}, +\infty)$

$$f(x) = \frac{x-2}{2-x^3} \qquad g(x) = (x+1)^2$$

I.D. $\mathbb{R}$

$$g \circ f(x) = g\left(\frac{x-2}{2-x^3}\right) = \left(\frac{x-2}{2-x^3} + 1\right)^2$$

$$f \circ g(x) = \frac{(x+1)^2 - 2}{2-(x+1)^6}$$

I.D.

$$(x+1)^6 \neq 2 \iff x+1 \neq \pm\sqrt[6]{2}$$

$$\iff x \neq -1 \pm \sqrt[6]{2}$$

1.28 (a voi 1.29)

$$f(x) = x^3 - 1 \qquad g(x) = \frac{2}{x+1} \quad \text{I.D. } x \neq -1$$

$$f \circ f(x) = f(x^3 - 1) = (x^3-1)^3 - 1 \quad \text{I.D. } \mathbb{R}$$

$$f \circ g(x) = f\left(\frac{2}{x+1}\right) = \left(\frac{2}{x+1}\right)^3 - 1 \quad \text{I.D. } x \neq -1$$

$$g \circ f(x) = g(x^3 - 1) = \frac{2}{(x^3-1)+1} = \frac{2}{x^3} \quad \text{I.D. } x \neq 0$$

$$g \circ g(x) = g\left(\frac{2}{x+1}\right) = \frac{2}{\frac{2}{x+1}+1} = \frac{2(x+1)}{2+x+1} =$$

$$= \frac{2(x+1)}{x+3}$$

Tracciare il grafico di questa LEGGE (altra pag)

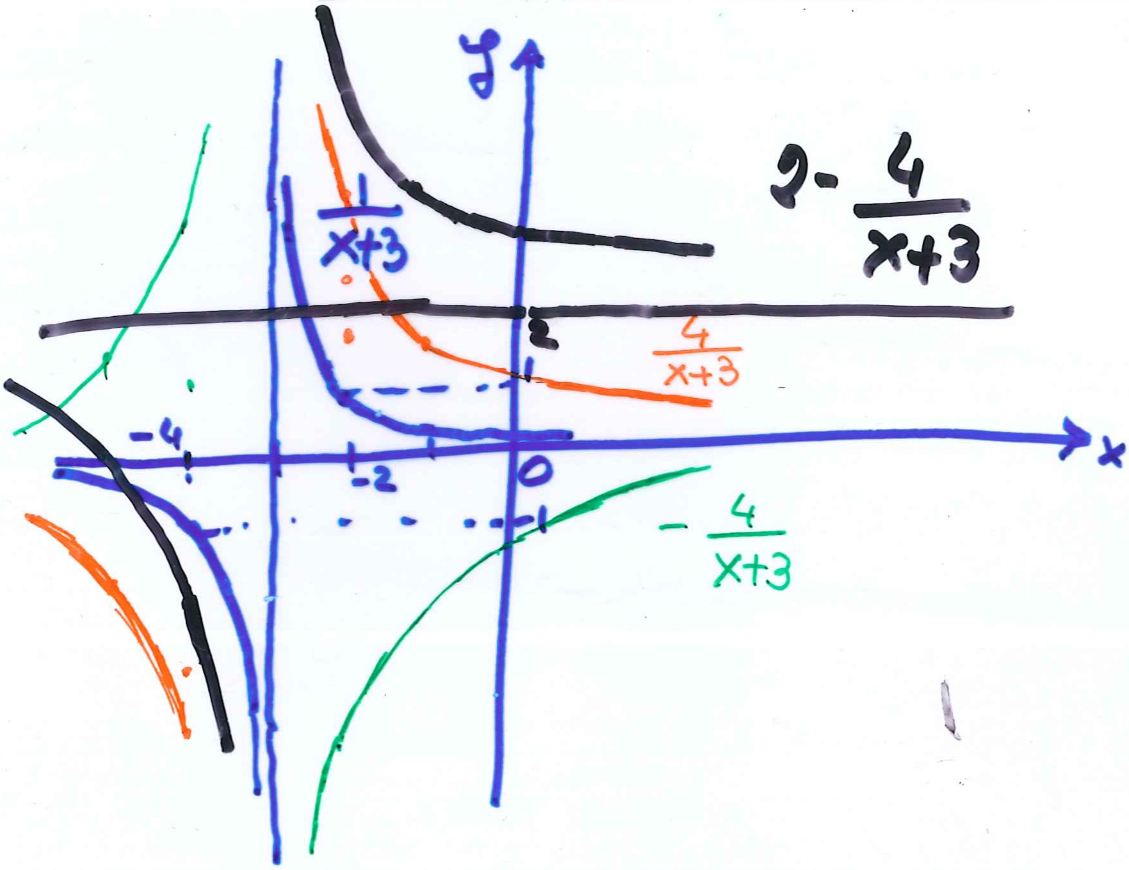
$$x \xmapsto{()+1} x+1 \xmapsto{\frac{1}{()}} \frac{1}{x+1} \xmapsto{2\cdot()} \frac{2}{x+1} \xmapsto{()+1} \frac{2}{x+1}+1 \xmapsto{\frac{1}{()}}$$

$$\longmapsto \frac{1}{\frac{2}{x+1}+1} \xmapsto{2()} \frac{2}{\frac{2}{x+1}+1}$$

$$\text{I.D.} \begin{cases} x \neq -1 \\ x \neq -3 \end{cases}$$

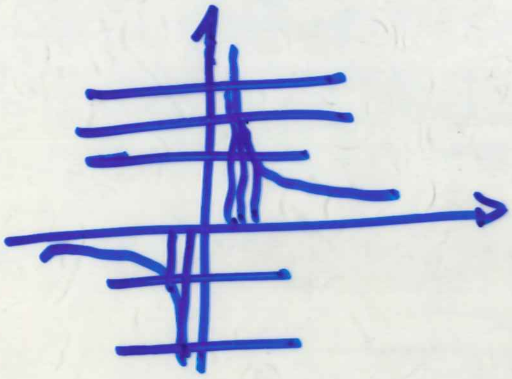
$$\frac{2(x+3)-4}{x+3} = 2 - \frac{4}{x+3}$$

è il modo per tracciare facilmente il grafico


y
x
0
-4
-2
2
1
-1
$\frac{1}{x+3}$
$2-\frac{4}{x+3}$
$\frac{4}{x+3}$
$-\frac{4}{x+3}$

$f(x) = \frac{1}{x}$: ha inversa? sì

Che forma ha?



$y = \frac{1}{x}$

$x = \frac{1}{y}$ $\qquad f^{-1}(y) = \frac{1}{y}$

$f(x) = x$ ha inversa? sì

$f^{-1}(y) = y.$

Scomporre

$f(x) = \frac{1}{(x-1)^2}$

Come se volessimo ad ~~es. calcolare~~:

$f(3)$

$3-1 = 2$

$(3-1)^2 = 4$

$\frac{1}{(3-1)^2} = \frac{1}{4}$

$x \xmapsto{F} x-1 \xmapsto{G} (x-1)^2 \xmapsto{H} \frac{1}{(x-1)^2}$

$y = F(x) = x-1$

$z = G(y) = y^2$

$H(z) = \frac{1}{z}$

$$x \xmapsto{(\ )-1} x-1 \xmapsto{(\ )^2} (x-1)^2 \xmapsto{\frac{1}{(\ )}} \frac{1}{(x-1)^2}$$

Scomporre:

$$f(x) = \frac{1}{\sqrt{2-x}-1}$$

I.D. $\begin{cases} x \leq 2 \\ 2-x \neq 1 \end{cases}$

$(-\infty, 1) \cup (1, 2]$

$$x \xmapsto[F]{-(\ )} -x \xmapsto[G]{(\ )+2} 2-x \xmapsto[H]{\sqrt{(\ )}} \sqrt{2-x} \xmapsto[K]{(\ )-1} \sqrt{2-x}-1 \xmapsto[L]{\frac{1}{(\ )}} \frac{1}{\sqrt{2-x}-1}$$

$y = F(x) = -x$

$z = G(y) = y+2$

$w = H(z) = \sqrt{z}$

$v = K(w) = w-1$

$L(v) = \frac{1}{v}$

$L \circ K \circ H \circ G \circ F(x)$