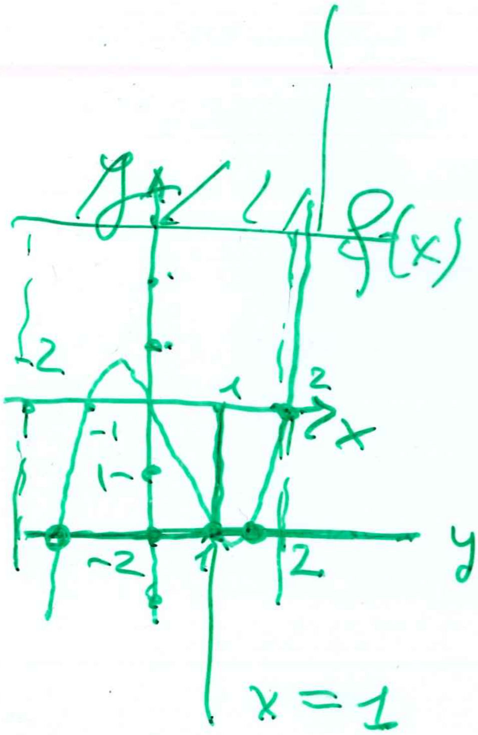


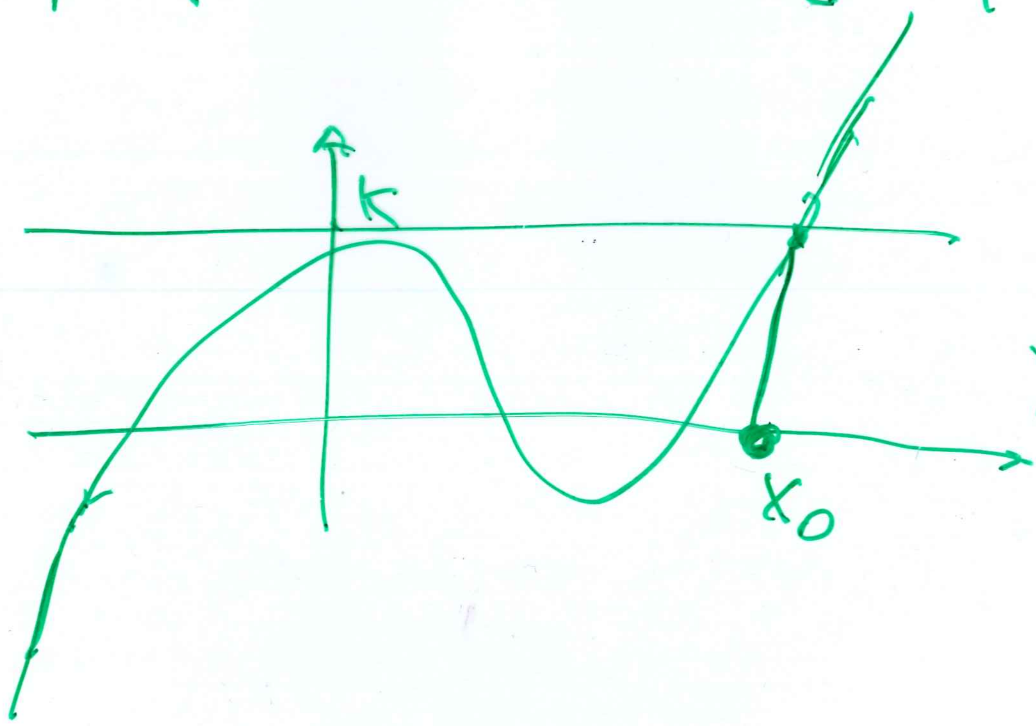
Es 1.14

$f(1) = ?$

in quanti punti di $[-2;2]$ dell'asse x la funzione assume il valore $f(1)$

$f(-1) = 2$

per quanti $x \in [-2, 2]$ $f(x) > 3$



$f(x) > K$

per $x > x_0$

2.2 (2)

$$|x^3 - x| \le 0$$

$$\Updownarrow \qquad \forall z \quad |z| \ge 0$$

$$|x^3 - x| = 0$$

$$\Updownarrow$$

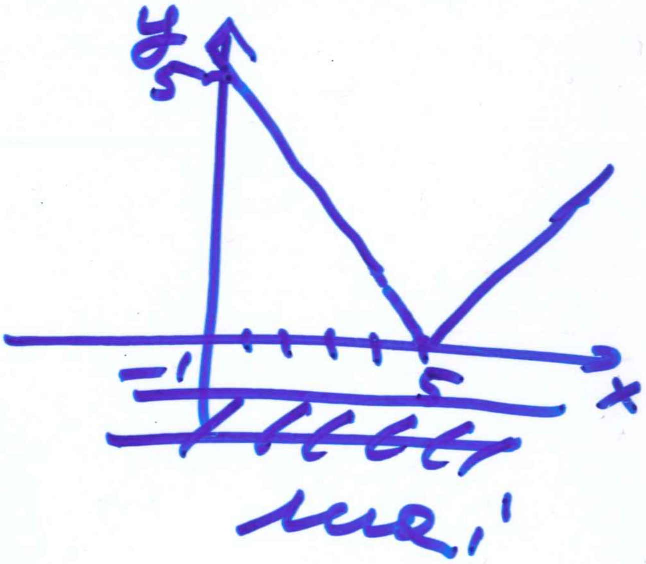
$$x^3 - x = 0$$

$$\Updownarrow$$

$$x(x^2 - 1) = 0 \iff x(x-1)(x+1) = 0$$

$$\boxed{x = 0, \pm 1}$$

$$|x - 5| \le -1$$



R: $\emptyset$

per nessun $x \in \mathbb{R}$

9.2(4)

2.2 (6)

$$\frac{|x-1|}{|5-x|} \leq 1$$

$|x-5| = \begin{cases} x-5 & \text{se } x>5 \\ 0 & \text{se } x=5 \\ 5-x & \text{se } x<5 \end{cases}$

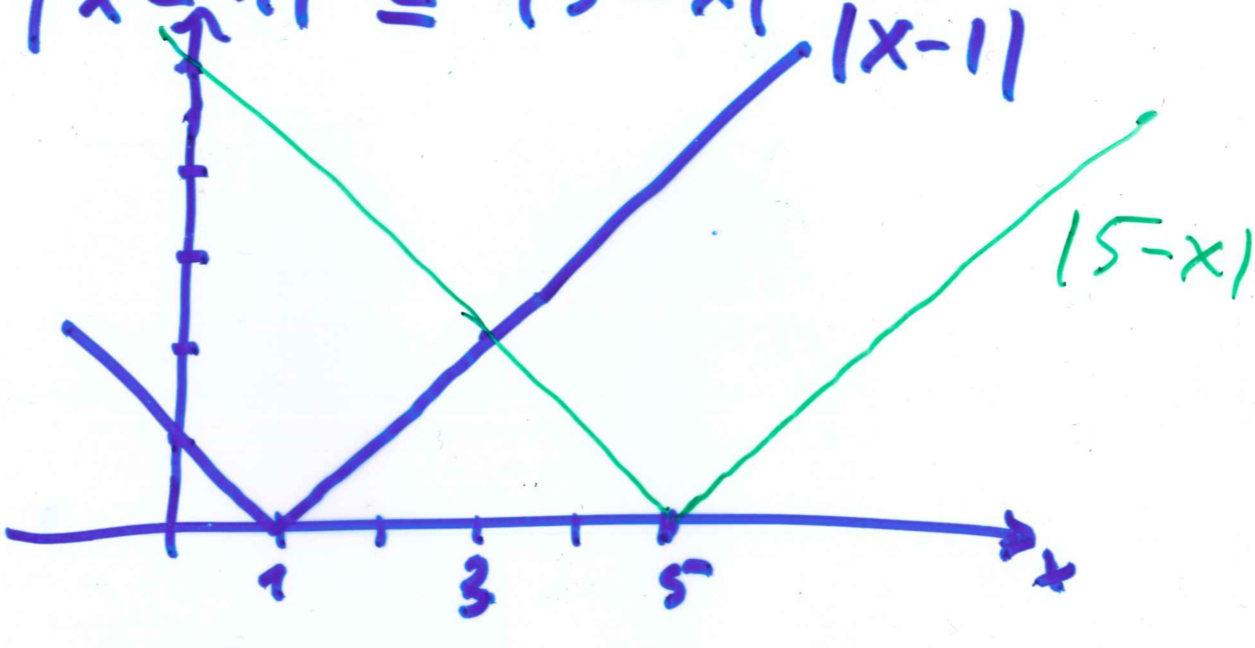
$|5-x| = \begin{cases} 5-x & \text{se } 5>x \\ 0 & \text{se } 5=x \\ x-5 & \text{se } 5<x \end{cases}$

I.D. $x \neq 5$.

moltiplico per $|x-5| = |5-x| > 0$

La disug. si conserva!

$|x-1| \leq |5-x|$



$x<3 \quad |x-1| < |5-x|$

$x=3 \quad |x-1| = |5-x|$

$x>3 \quad |x-1| > |5-x|$

algebricamente:

$$|x-1| \leq |x-5|$$

$x \in (-\infty, 1]$: $1-x \leq 5-x$

$x \in (1, 5]$: $x-1 \leq 5-x$

$x \in (5, +\infty)$: $\cancel{x}-1 \leq \cancel{x}-5$

3 casi da unire, NON SISTEMA; il sistema è tra la condizione $x \in \ldots$ e la disuguaglianza

$x \in (-\infty, 1]$: $1 \leq 5$ sempre: Sol $(-\infty, 1]$

$x \in (1, 5]$: $2x \leq 6$ $\quad \begin{cases} x \leq 3 \\ 1 < x \leq 5 \end{cases}$

Sol. : $(1, 3]$

$x \in (5, +\infty)$: $\emptyset$ e quindi

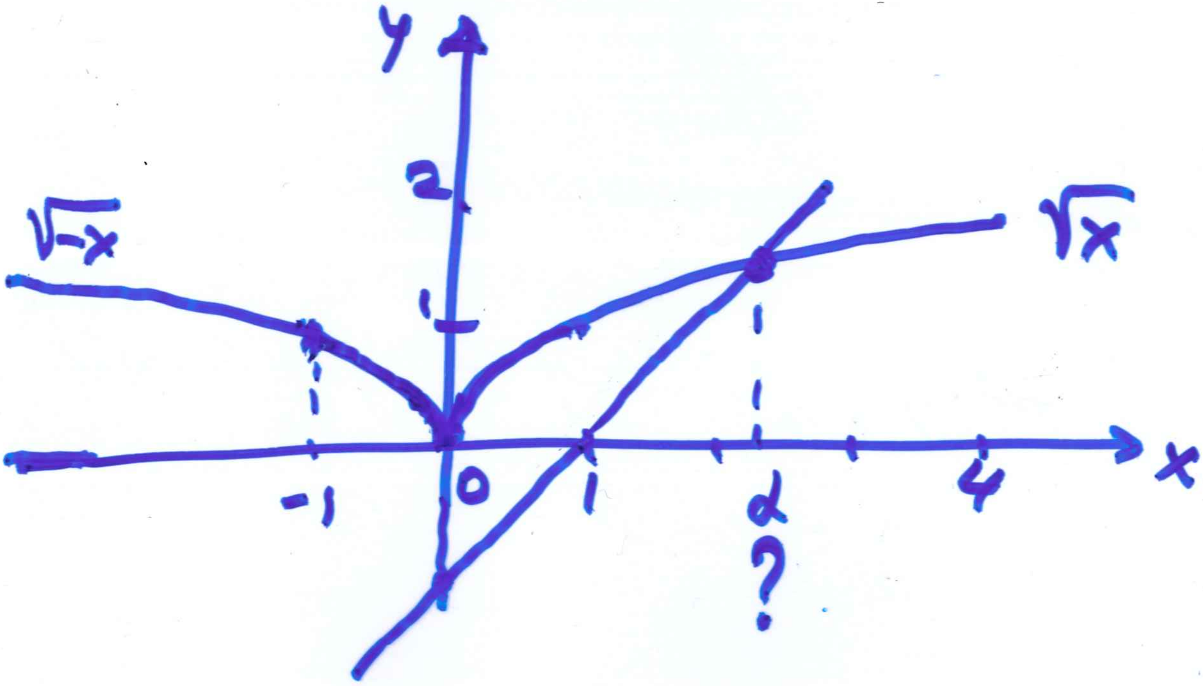
$\emptyset \cap (5, +\infty) = \emptyset$

$$(-\infty, 1] \cup (1, 3] = (-\infty, 3]$$

2.5 (1)

$$\sqrt{|x|} > x-1$$

I.D. $|x| \geq 0 \iff \forall x \in \mathbb{R}$



$\forall x < \alpha \quad \sqrt{|x|} > x-1$

Se $x \leq 1$ 2° membro ≤ 0
$\Rightarrow$ è minore di $\sqrt{|x|}$

Se $x > 1$ $\quad |x| > (x-1)^2$ $\quad x = \frac{3 \pm \sqrt{5}}{2}$

$$\begin{cases} x > 1 \\ x > x^2 - 2x + 1 \end{cases} \qquad \begin{cases} x > 1 \\ x^2 - 3x + 1 < 0 \end{cases}$$

$$\begin{cases} x > 1 \\ \dfrac{3-\sqrt{5}}{2} < x < \dfrac{3+\sqrt{5}}{2} \end{cases} \qquad 1 < x < \frac{3+\sqrt{5}}{2}$$

Unito all'altro intervallo $(-\infty, 1)$:

Sol : $\left(-\infty, \dfrac{3+\sqrt{5}}{2}\right)$

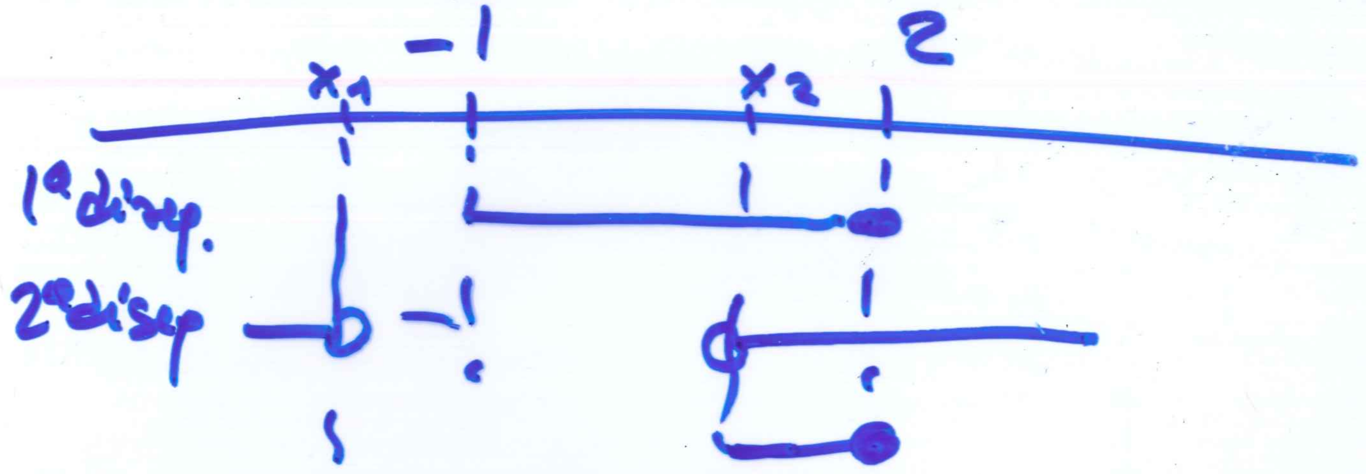
$$\sqrt{4-x^2} < x+1$$

$$\begin{cases} 4-x^2 \geq 0 & \text{radicando} \geq 0 \\ x+1 > 0 & \text{II membro} > \sqrt{\ } \geq 0 \\ 4-x^2 < (x+1)^2 \end{cases}$$

$$\begin{cases} -2 \leq x \leq 2 \\ x > -1 \\ 2x^2+2x-3 > 0 \end{cases} \qquad \begin{cases} -1 < x \leq 2 \\ x < \dfrac{-1-\sqrt{7}}{2} \vee x > \dfrac{-1+\sqrt{5}}{2} \end{cases}$$

$$2x^2+2x-3=0 \qquad x_{1,2} = \frac{-1 \pm \sqrt{1+6}}{2} =$$

$-1 < x_2 < 2 \implies$ Sol $\left(\dfrac{\sqrt{7}-1}{2}, 2\right]$



(3)

$$x\sqrt{3-2x} \le 1$$

I.D. $3-2x \ge 0 \Rightarrow x \le 3/2$

se $x \le 0$ $\quad x\sqrt{3-2x} \le 0 < 1$

se $0 < x \le \frac{3}{2}$: $\quad x^2(3-2x) \le 1$

$$2x^3 - 3x^2 + 1 \ge 0$$

$2\cdot 1 - 3\cdot 1 + 1 = 0 \quad x = 1$ è sol.!

$$\begin{array}{c|ccc|c} & 2 & -3 & 0 & 1 \\ 1 & & 2 & -1 & -1 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$$2x^3 - 3x^2 + 1 = (x-1)(2x^2 - x - 1) =$$

$= (x-1)^2 (2x+1) \geq 0$

$\left.\begin{array}{l} 0 \text{ per } x = 1 \\ 0 \text{ per } x \geq -\frac{1}{2} \end{array}\right\}$

quindi $2x^3 - 3x^2 + 1 \geq 0$ per

$x \in [-1/2, +\infty)$

$\begin{cases} 2x^3 - 3x^2 + 1 \geq 0 \\ 0 < x \leq 3/2 \end{cases}$ Sol: $(0, 3/2]$

Concludendo la disug. vale in $(-\infty, 3/2]$

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(10)

$x - 3 < \sqrt[3]{x^3 - x}$ I.D. $\mathbb{R}$

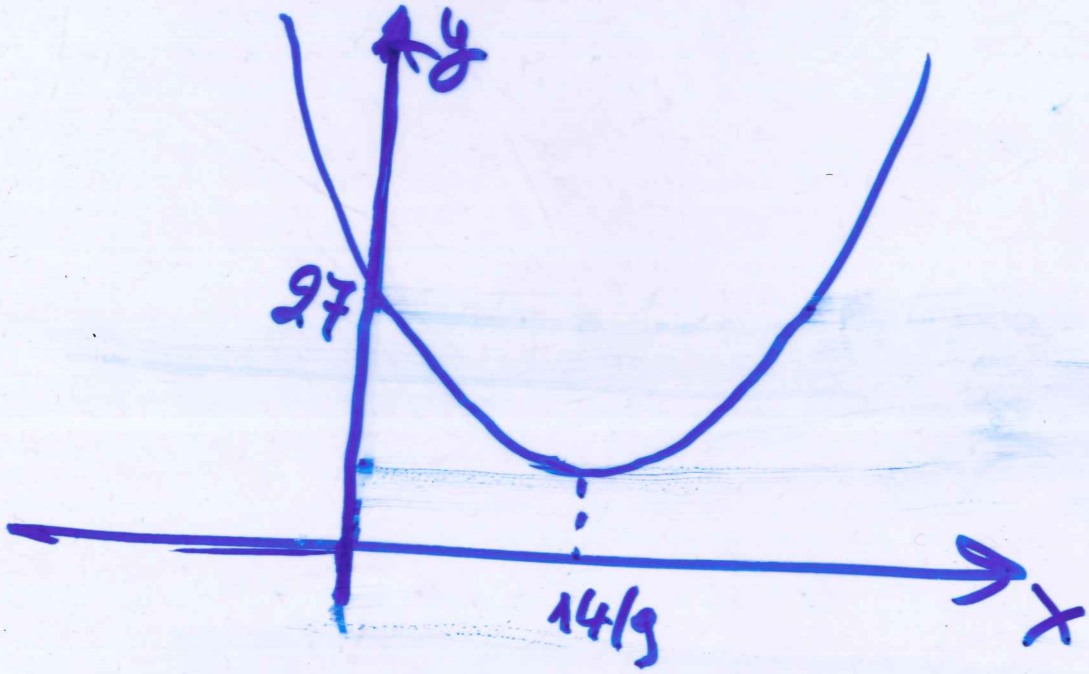
$\downarrow y^3$: conserva il verso

$(x-3)^3 < x^3 - x$

$\cancel{x^3} - 3x^2(+3) + 3x(+3)^2 - 3^3 < \cancel{x^3} - x$

$$9x^2 - 28x + 27 > 0 \qquad \boxed{\forall x \text{ poiché}}$$

$$x_{1,2} = \frac{+14 \pm \sqrt{14^2 - 27\cdot 9}}{9} \leftarrow \Delta < 0$$



Corrisponde al fatto algebrico che il polinomio si può leggere come somma di un quadrato e di un numero > 0

$$\left(3x + \frac{14}{3}\right)^2 + \frac{49}{9} > 0$$

Invece:

$$-9x^2 + 28x - 27 =$$

$$-\left(3x + \frac{14}{3}\right)^2 - \frac{49}{9} < 0$$

2.6, 2.8 Compito.