35.1 k)

$$\lim_{n\to\infty} \frac{n-3^{-n}}{\ln\left(1+3^{n}\right)+n^{2/3}} = \frac{\infty - 0}{\infty+\infty} \quad \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{n\to\infty} \frac{n}{\ln\left(3^{n}\left(1+3^{-n}\right)\right)+n^{2/3}} =$$

$$= \lim_{n\to\infty} \frac{n}{\ln 3^{n} + n^{2/3} + \underbrace{\ln\left(\underbrace{1+3^{-n}}_{\to 1}\right)}_{\to 0}} =$$

$$= \lim_{n\to\infty} \frac{n}{n\ln 3 + n^{2/3}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\ln 3\underbrace{\left(1+\frac{1}{n^{1/3}\ln 3}\right)}_{\to 1}} = \frac{1}{\ln 3}$$

$$\lim \frac{n}{\ln(\underbrace{1+n+n^{7/2}}+3^n)} =$$

$$= \lim \frac{n}{\ln\left(3^n\cdot\left(1+\frac{1}{3^n}+\frac{n}{3^n}+\frac{n^{7/2}}{3^n}\right)\right)} =$$

$$= \lim \frac{n}{n\ln 3 + \underbrace{\ln\underbrace{\left(1+\underbrace{\frac{1+n+n^{7/2}}{3^n}}_{\to 0}\right)}_{\to 1}}_{\to 0}}$$

$\ln(1+b_n) \sim b_n$ se $\{b_n\} \to 0$

dice che

$\ln\left(1+\dfrac{1+n+n^{7/2}}{3^n}\right)$ tende a 0

Come $\dfrac{1+n+n^{7/2}}{3^n}$

ES 35.2 b)

$$\lim_{n\to\infty} n^2 \operatorname{sen}\left(\frac{3n+1}{n^2}\right) = [\infty \cdot 0] =$$

$$\boxed{\begin{array}{l} \text{visto che } \left\{\frac{3n+1}{n^2}\right\} \longrightarrow 0 \\ \operatorname{sen}\left(\frac{3n+1}{n^2}\right) \sim \frac{3n+1}{n^2} \end{array}}$$

$$= \lim_{n\to\infty} \cancel{n^2} \cdot \frac{3n+1}{\cancel{n^2}} = +\infty$$

35.3 b)

$$\lim_{n\to\infty}\left(\frac{n^2+n}{n^2-n+1}\right)^{-n} = [1^\infty]$$

$$= \lim_{n\to\infty}\left(1+\frac{2n-1}{n^2-n+1}\right)^{-n} =$$

$$= \lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{-n} = e^{-2}$$

(3s.4 d) 4

$$\lim_{n\to\infty}\left(\sqrt[3]{\frac{2n-1}{2n+2}}\right)^n = [1^\infty]$$

$$= \lim_{n\to\infty}\left(\frac{2n-1}{2n+2}\right)^{n/3} =$$

$$= \lim_{n\to\infty}\left(1+\frac{-3}{2n+2}\right)^{n/3} =$$

$$= \lim\left(1+\frac{1}{-\frac{2}{3}n}\right)^{n/3} =$$

$$= \lim\left(1+\frac{1}{-\frac{2}{3}n}\right)^{\left(-\frac{2}{3}n\right)\left(-\frac{1}{2}\right)} = e^{-1/2}$$

3s.5 a)

$$\lim_{n\to+\infty} n\ln\left(\frac{n+3}{n}\right) = [\infty\cdot 0]$$

$$= \lim_{n\to+\infty} \ln\left(1+\frac{3}{n}\right)^n = 3$$

Oppure

$\ln\left(1+\frac{3}{n}\right) \sim \frac{3}{n}$ poiché $\frac{3}{n}\to 0$

$$\lim_{n\to\infty} n\ln\left(\frac{n+3}{n}\right) = \lim_{n\to\infty} n\cdot\frac{3}{n} = 3$$

35.6 c)

$$\lim_{n\to\infty} e^{n+1} \ln(1+e^{-n}) = [\infty \cdot 0] = e$$

$$\boxed{\{e^{-n}\} \to 0 \Rightarrow \ln(1+e^{-n}) \sim e^{-n}}$$

35.7 d)

$$\lim_{n\to\infty} \sqrt{n^2+5n} - \sqrt{n^2+3n} = [\infty - \infty]$$

$$= \lim_{n\to\infty} \frac{(n^2+5n)-(n^2+3n)}{\sqrt{n^2+5n}+\sqrt{n^2+3n}} =$$

$$= \lim_{n\to\infty} \frac{2n}{\sqrt{n^2}+\sqrt{n^2}} = \frac{2n}{2n} = 1$$

$$\sqrt{n^2} = |n| = n$$

Oppure

$(1+b_n)^t = 1 + tb_n + o(b_n)$ se $\{b_n\} \to 0$

$$\lim_{n\to\infty} n\left(\sqrt{1+5/n} - \sqrt{1+3/n}\right) =$$

$$\lim_{n\to\infty} n\left[\left(\cancel{1}+\frac{1}{2}\cdot\frac{5}{n}+o\left(\frac{1}{n}\right)\right)-\left(\cancel{1}+\frac{1}{2}\cdot\frac{3}{n}+o\left(\frac{1}{n}\right)\right)\right]$$

$$= \lim \left(n\left(\frac{1}{n}+o\left(\frac{1}{n}\right)\right)\right) = 1+o(1) \to 1$$

35.8 a)

$$\lim_{n\to\infty} \ln(3n^3-2n) - \ln(n^3+1) = \boxed{\infty-\infty}$$

$$= \lim_{n\to\infty} \ln \frac{3n^3-2n}{n^3+1} =$$

$$= \lim \ln \frac{3n^3(1-2/3n^2)}{n^3(1+1/n^3)} =$$

$$= \ln 3 \left(\frac{1-2/3n^2}{1+1/n^3}\right) = \ln 3$$

$$= \lim \ln 3 \boxed{\frac{n^3-2/3n}{n^3+1}} =$$

$$= \lim \ln 3 + \underbrace{\ln \frac{n^3-2/3n}{n^3+1}}_{\to 0} =$$

$$= \ln 3$$

35.8 d

$$\lim_{u\to\infty} \sqrt[3]{2u^3+u^2+1} + \sqrt[3]{3u-2u^3} =$$

$$= \infty + (-\infty) \quad [\infty - \infty]$$

$$= \lim_{u\to\infty} \sqrt[3]{2u^3 \underline{+u^2+1}} - \sqrt[3]{2u^3 \underline{-3u}}$$

$$\boxed{a - b = \frac{a^3 - b^3}{a^2+ab+b^2}}$$

$2u^3+u^2+1 =$
$2u^3 + o(u^3)$
ecc.

$$= \lim_{u\to\infty} \frac{\cancel{2u^3}+u^2+1-(\cancel{2u^3}-3u)}{\left(\sqrt[3]{2u^3}\right)^2 + \sqrt[3]{2u^3}\cdot\sqrt[3]{2u^3} + \left(\sqrt[3]{2u^3}\right)^2} =$$

$$= \lim_{u\to\infty} \frac{u^2+3u+1}{3\,(2u^3)^{2/3}} =$$

$$= \lim \frac{u^2+\cancel{o(u^2)}}{3\cdot 2^{2/3}\cdot u^2} = \frac{1}{3\cdot 2^{2/3}}$$

35.5 b)

$$\lim_{n\to\infty} n\left(1-3^{1/n}\right)=[\infty\cdot 0]$$

$$=\lim_{n\to\infty} -n\left(e^{(\ln 3)\cdot\frac{1}{n}}-1\right)=$$

$$=\lim_{n\to\infty} -\cancel{n}\cdot \ln 3\cdot\frac{1}{\cancel{n}}=-\ln 3$$

$$\text{si } \{b_n\}\to 0;\ \left\{\frac{e^{b_n}-1}{b_n}\right\}\to 1$$

35.2 f)

$$\lim_{n\to\infty}\frac{(\cos\sqrt{n})-1}{n}=0-0=0$$

35. d)

$$\lim_{n\to\infty} n\left(\cos\frac{1}{\sqrt{n}}-1\right)=[\infty\cdot 0]$$

$$=\lim_{n\to\infty} n\cdot\frac{\cos^2(1/\sqrt{n})-1}{\cos(1/\sqrt{n})+1}=\quad -1/2$$

$$\shortparallel$$

$$=\lim_{n\to\infty} n\cdot\frac{-\operatorname{sen}^2(1/\sqrt{n})}{\cos(1/\sqrt{n})+1}=\lim\frac{n\left[-\left(1/\sqrt{n}\right)^2\right]}{\cos\frac{1}{\sqrt{n}}+1}$$

3s.9 c)

$$\lim_{n\to\infty}\left(e^{n}+2\right)^{1/n} = [\infty^0] =$$

$$= \lim_{n\to\infty} e^{\frac{1}{n}\ln\left(e^{n}+2\right)} = e^{\lim \frac{\ln(e^{n}+2)}{n}} = e$$

$$\lim \frac{\ln(e^{n}+2)}{n} = \lim \frac{\ln\left(e^{n}\left(1+2e^{-n}\right)\right)}{n} =$$

$$= \lim \frac{\ln e^{n} + \underbrace{\ln\left(1+2e^{-n}\right)}_{\to 0}}{n}$$

$$= \lim \frac{\cancel{n}\ln e}{\cancel{n}} = 1$$

3s.10a

$$\lim_{n\to\infty}\left(\operatorname{sen}\frac{1}{n}\right)^{3/\sqrt{n}} = [0^0]$$

$$\frac{-3\ln n}{\sqrt{n}} = -3\frac{\ln n}{n^{1/2}}$$

$$= \lim e^{\frac{3\ln(\operatorname{sen} 1/n)}{\sqrt{n}}} = e^{\lim \frac{3\ln(1/n)}{\sqrt{n}}} = e^{\lim \frac{-3\ln n}{\sqrt{n}}}$$

$$= e^{0^-} = 1^-$$

10) 3S.10 d)

$$\lim_{n\to\infty}\left(\sqrt{e^{-n}+1}-1\right)^{1/n}=[0^0]=$$

$$\left(\sqrt{1+b_n}-1\right)^{1/n} \qquad \text{coll}\{b_n\}\to 0$$

$$\Big(\underbrace{(1+b_n)^{1/2}-1}_{\sim\frac{1}{2}b_n}\Big)^{1/n} \sim \left(\frac{1}{2}b_n\right)^{1/n}=$$

$$\boxed{(1+b_n)^t=1+t\,b_n+\\+o(b_n)}$$

$$=\left(\frac{1}{2}\right)^{1/n}\cdot e^{-\frac{n}{n}}=e^{-1}$$

oppure

$$=\lim e^{\frac{\ln\left(\sqrt{e^{-n}+1}-1\right)}{n}}=e^{\lim\frac{\ln 1/2-n}{n}}$$

$$\ln\left(\frac{\sqrt{e^{-n}+1}-1}{e^{-n}}\cdot e^{-n}\right)=$$

$$=\underbrace{\ln\Big(\underbrace{\frac{\sqrt{e^{-n}+1}-1}{e^{-n}}}_{\to 1/2}\Big)}_{\to\ln 1/2}+\underbrace{\ln e^{-n}}_{=-n}$$

$$\left(\sqrt[n]{n-1}\right)^{\ln n}$$

$$\lim_{n\to\infty} (n-1)^{\frac{\ln n}{n}} = [\infty^0] =$$

$$= \lim_{n\to\infty} e^{\ln(n-1)\cdot\frac{\ln n}{n}} =$$

$$= e^{\lim \frac{\ln(n-1)\cdot\ln n}{n}} =$$

$$= e^{\lim \frac{(\ln n)^2}{n}} = e^0 = 1$$

$$\left\{\sin\left(\frac{n^2-3n^3}{n^3}\right)\right\} \sim \left\{\sin\frac{-3n^3}{n^3}\right\} \to$$

$$\to -\sin 3$$

$$\left\{\frac{\sin(n^2-3n^3)}{n^3}\right\} \longrightarrow 0$$

$$\lim_{n\to\infty} \frac{1}{n}\cdot\Big(\underbrace{\ln(1+n^2)}_{+\infty} + \underbrace{e^{-2n}}_{\downarrow\, 0}\Big) =$$

$$= \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot \ln(1+n^2) =$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot 2\ln n = 0.$$

$$\lim_{n\to+\infty} \frac{\operatorname{sen}\left(2/n^2 + 1/n\right)}{10/n} = \left[\frac{0}{0}\right]$$

tra l'infinitesimo $\frac{2}{n^2}$ e $\frac{1}{n}$ quale posso trascurare?

$$\frac{2}{n^2}$$

$$\lim_{n\to+\infty} \frac{\operatorname{sen}(1/n)}{10/n} = \frac{1}{10}$$