ESERCIZI su primitive

$\boxed{\text{PG N 16}}$

I.D. $x \neq k\pi$, $k \in \mathbb{Z}$
continuità: su ogni intervallo
$\left(-\frac{\pi}{2}+k\pi, \frac{\pi}{2}+k\pi\right)$, ove quindi sono def. le primitive.

$$\int \frac{dx}{\operatorname{sen} x} = \boxed{x = 2t, \quad dx = 2\,dt}$$

$$= \int \frac{2\,dt}{\operatorname{sen} 2t} = \int \frac{\cancel{2}\,dt}{\cancel{2}\operatorname{sen} t \cos t} =$$

$$= \int \frac{(\cos t)^2 + (\operatorname{sen} t)^2}{\operatorname{sen} t \cos t}\,dt =$$

$$= \int \left(\frac{\cos t}{\operatorname{sen} t} + \frac{\operatorname{sen} t}{\cos t}\right) dt =$$

$$= \ln|\operatorname{sen} t| - \ln|\cos t| + c = \boxed{t = \frac{x}{2}}$$

$$= \ln\left|\frac{\operatorname{sen} x/2}{\cos x/2}\right| + c$$

$\boxed{\text{P6} \quad \text{N17}}$

(continua in $(k\pi, (k+1)\pi)$, $k \in \mathbb{Z}$)

$$\int \frac{1}{\cos x}\, dx =$$

$\cos x = \operatorname{sen}(\pi/2 - x)$

$t = \frac{\pi}{2} - x$

$dt = -dx$

$$\Downarrow$$

$$\int \frac{-dt}{\operatorname{sen} t} =$$

$$= -\ln \left| \frac{\operatorname{sen} t/2}{\cos t/2} \right| + c =$$

$$= \ln \left| \frac{\cos(\pi/4 - x/2)}{\operatorname{sen}(\pi/4 - x/2)} \right| + c$$

$$\int \frac{1}{x\sqrt{x-1}}\, dx =$$

$$\boxed{\begin{aligned} \sqrt{x-1} &= t \\ x-1 &= t^2 \\ dx &= 2t\,dt \end{aligned}}$$

$$= \int \frac{2\cancel{t}\,dt}{(1+t^2)\,\cancel{t}} = 2\arctan t + c =$$

$$= 2\arctan\left(\sqrt{x-1}\right) + c$$

Attenzione all'ID dell'integranda: $(1, +\infty)$. Qui essa è continua e quindi ogni primitiva è definita in $(1, +\infty)$

ID integranda: $(-1,0)\cup(0,+\infty) \Rightarrow$ continua su $(-1,0)$ e su $(0,+\infty)$

$$\int \frac{1}{x\sqrt{x+1}} \qquad \boxed{\begin{aligned}\sqrt{x+1} &= t\\ x+1 &= t^2\\ dx &= 2t\,dt\end{aligned}}$$

(3)

$$= \int \frac{2t\,dt}{t(t^2-1)} = 2\int \frac{dt}{t^2-1} =$$

$\forall t$ t.c.

$$\frac{1}{t^2-1} = \frac{1}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$$

$$= \frac{(A+B)t + A - B}{t^2-1} \Rightarrow$$

$$\begin{cases} A+B=0 \\ A-B=1 \end{cases} \quad \begin{cases} B=-1/2 \\ A=1/2 \end{cases}$$

$$= 2\cdot\frac{1}{2}\int\left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt =$$

$$= \ln\left|\frac{t-1}{t+1}\right| + c = \ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + c$$

La primitiva trovata ha due possibili intervalli di definizione:

$(0,+\infty)$ e $(-1,0)$

ESERCIZI su Aree

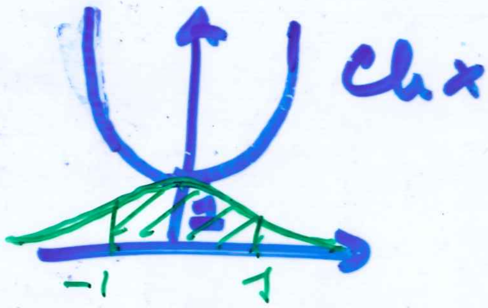
I 7 ES. (4

1. Area del trapezoide delimitato da asse x e grafico di

$$f(x) = \frac{2}{e^{x}+e^{-x}} \quad \text{in } [-1, 1]$$

Svolg.

$$f(x) = \frac{1}{\mathrm{Ch}\, x}$$



$f(x) > 0$ in $[-1, 1]$ $\Rightarrow$

$$\text{Area} = \int_{-1}^{1} f(x)\,dx \underset{\text{per la parità di } f(x)}{=} 2\int_{0}^{1} f(x)\,dx$$

Calcolo l'integr. indefinito

$$\int \frac{2}{e^{x}+e^{-x}}\,dx = \int \frac{2\,dx}{e^{-x}(e^{2x}+1)} = \int \frac{2e^{x}dx}{e^{2x}+1} =$$

$$\boxed{\begin{array}{l} e^{x}=t \\ e^{x}dx=dt \\ e^{2x}=t^{2} \end{array}} = \int \frac{2\,dt}{1+t^{2}} = 2\,\mathrm{arctg}\, e^{x} + c$$

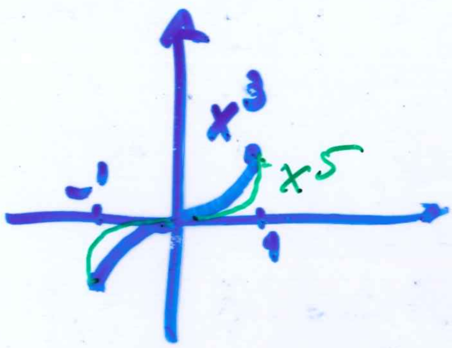
$$\text{Area} = 2\left[\arctan e^{x}\right]_{0}^{1} = 4\left(\arctan e - \frac{\pi}{4}\right)$$

2. Calcolare l'area della regione di piano compresa tra i grafici di $f(x)=x^3$, $g(x)=x^5$ nell'intervallo $[-1,1]$. .... E in $[-1,2]$?

f e g sono dispari

in $[0,1]$: $f(x) \geq g(x)$

in $[-1,0]$: $f(x) \leq g(x)$



Area $= 2$ Area tra $\underline{0 \text{ e } 1}$

$$= 2\int_0^1 (x^3 - x^5)\,dx =$$

$$= 2\left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^1 =$$

$$= 2\left(\frac{1}{4} - \frac{1}{6}\right) = \frac{1}{6}.$$

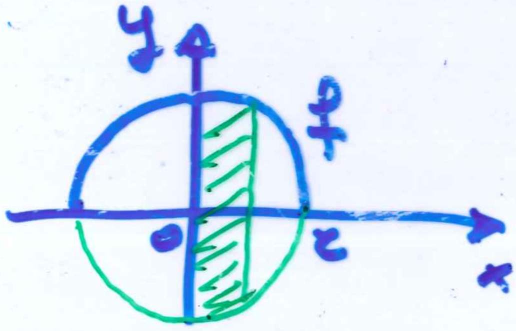
In $[-1,2]$ cambia che si deve sommare all'area precedente $\int_1^2 (g(x)-f(x))\,dx =$ se $x>1$ $\boxed{g(x) > f(x)}$

$$= \left[\frac{x^6}{6} - \frac{x^4}{4}\right]_1^2 = \left(\frac{64}{6} - \frac{16}{4} - \frac{1}{6} + \frac{1}{4}\right)$$

Area di R ove R è la regione compresa tra $f(x)=\sqrt{r^2-x^2}$

$g(x)=-\sqrt{r^2-x^2}$ } semicirconferenza di centro (0,0) e raggio r

e $x=0$, $x=\frac{r}{2}$



$$\text{Area}=2\int_0^{r/2} f(x)\,dx=$$

$$=2r^2\left(\frac{\pi}{12}+\frac{\sqrt{3}}{8}\right)=\dots$$

$$\int\sqrt{r^2-x^2}\,dx=\begin{cases} x=r\cos t & t\in[0,\pi] \\ \sqrt{r^2-x^2}=r\,\mathrm{sen}\,t \\ dx=-r\,\mathrm{sen}\,t\,dt \end{cases}$$

$$=\int -r^2\,\mathrm{sen}^2 t\,dt=r^2\cdot\frac{-t+\mathrm{sen}\,t\cos t}{2}+c=$$

$$=\frac{r^2}{2}\left(-\arccos\frac{x}{r}+\frac{x}{r}\,\frac{\sqrt{r^2-x^2}}{r}\right)+c$$

usando la primitiva calcolata sopra

$$2\left[-\frac{r^2}{2}\arccos\frac{x}{r}+\frac{1}{2}x\sqrt{r^2-x^2}\right]_0^{r/2}=$$

$$=2\left(-\frac{r^2}{2}\,\frac{\pi}{3}+\frac{1}{2}\cdot\frac{r^2}{2}\,\frac{\sqrt{3}}{2}+\frac{r^2}{2}\cdot\frac{\pi}{2}\right)=\text{VEDI SOPRA}$$