Individuare e Studiare i punti critici

(1) $f(x,y) = (y - x^3)(y - x) =$

$$= y^2 - x^3 y - xy + x^4$$

$$f_x(x,y) = -3x^2 y - y + 4x^3$$

$$f_y(x,y) = 2y - x^3 - x$$

$$\text{grad } f = (4x^3 - 3x^2 y - y, -x^3 - x + 2y) = (0,0)$$

$$\begin{cases} y = \frac{1}{2}(x^3 + x) \\ 4x^3 - \frac{1}{2}(3x^2+1)(x^3+x) = 0 \end{cases}$$

$$\begin{cases} y = \frac{1}{2}(x^3 + x) \\ x\left(4x^2 - \frac{1}{2}(3x^4 + 4x^2 + 1)\right) = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ y = 0 \end{cases} \qquad \begin{cases} 3x^4 + 4x^2 + 1 - 8x^2 = 0 \\ y = \frac{1}{2}(x^3 + x) \end{cases}$$

$\Rightarrow P_0 = (0,0)$

$$\begin{cases} (3x^2 - 1)(x^2 - 1) = 0 \\ y = \frac{1}{2}(x^3 + x) \end{cases} \qquad \begin{cases} x = \pm\frac{1}{\sqrt{3}} \\ y = \frac{1}{2}\left(\pm\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3} + 1\right) \end{cases} \vee \begin{cases} x = \pm 1 \\ y = \pm 1 \end{cases}$$

I punti critici sono

$P_0 = (0,0)$, $P_1 = \left(\frac{1}{\sqrt{3}}, \frac{2}{3\sqrt{3}}\right)$, $P_2 = \left(\frac{-1}{\sqrt{3}}, \frac{-2}{3\sqrt{3}}\right)$

$P_3 = (1,1)$, $P_4 = (-1,-1)$

Studio

$f_{xx} = 12x^2 - 6xy$ $\qquad f_{xy} = -3x^2 - 1$

$f_{yx} = -3x^2 - 1$ $\qquad f_{yy} = 2$

$$H = \begin{vmatrix} 12x^2 - 6xy & -3x^2 - 1 \\ -3x^2 - 1 & 2 \end{vmatrix}$$

$$H(P_0) = \begin{vmatrix} 0 & -1 \\ -1 & 2 \end{vmatrix} < 0 \Rightarrow P_0 \text{ pto di sella}$$

$$H(P_1) = \begin{vmatrix} 12 \cdot \frac{1}{3} - 6 \cdot \frac{1}{\sqrt{3}} \cdot \frac{2}{3\sqrt{3}} & -3 \cdot \frac{1}{3} - 1 \\ -3 \cdot \frac{1}{3} - 1 & 2 \end{vmatrix} =$$

$$= \begin{vmatrix} 4 - \frac{4}{3} & -2 \\ -2 & 2 \end{vmatrix} = \frac{16}{3} - 4 > 0$$

P_1 min. locale forte

$H(P_2) = H(P_1)$

P_2 idem.

$$H(P_3) = \begin{vmatrix} 6 & -4 \\ -4 & 2 \end{vmatrix} = 12 - 16 < 0$$ P_3 pto di sella

$$H(P_4) = H(P_3)$$ P_4 punto di sella

$$f(x,y) = (x^2 - 2x)(y + x - 1)$$

I.D. $\mathbb{R}^2$; $f, f_x, f_y, f_{xx}, f_{xy}, f_{yx}, f_{yy}$ sono continue su $\mathbb{R}^2$ perché sono tutti polinomi.

$$f(x,y) = x^3 + x^2y - 3x^2 - 2xy + 2x$$

$$f_x = 3x^2 + 2xy - 6x - 2y + 2$$

$$f_y = x^2 - 2x$$

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \iff \begin{cases} x(x-2) = 0 \\ 3x^2 + 2xy - 6x - 2y + 2 = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ -2y + 2 = 0 \end{cases} \qquad \begin{cases} x = 2 \\ \cancel{12} + 4y - \cancel{12} - 2y + 2 = 0 \end{cases}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{cases} x = 0 \\ y = 1 \end{cases} \qquad \begin{cases} x = 2 \\ y = -1 \end{cases}$$

$P_1 = (0, 1)$ $\qquad$ $P_2 = (2, -1)$

$f_{xx} = 6x + 2y - 6$ $\qquad$ $f_{xy} = 2x - 2$

$f_{yx} = 2x - 2$ $\qquad$ $f_{yy} = 0$

$$H = \begin{vmatrix} 6x+2y-6 & 2(x-1) \\ 2(x-1) & 0 \end{vmatrix}$$

$$H(P_1) = \begin{vmatrix} -4 & -2 \\ -2 & 0 \end{vmatrix} < 0 \quad P_1 \text{ sella}$$

$$H(P_2) = \begin{vmatrix} 4 & 4 \\ 4 & 0 \end{vmatrix} < 0 \quad P_2 \text{ sella}$$

$f(x,y) = y^2 - (x^2-1)^2$

I.D $\mathbb{R}^2$. f, f_x, f_y, f_{xx}, f_{xy}, f_{yx}, f_{yy} cont. in $\mathbb{R}^2$ . . .

$f_x = -2(x^2-1)\cdot 2x$

$f_y = 2y$

punti critici $\begin{cases} x(x^2-1)=0 \\ y=0 \end{cases}$

$P_0 = (0,0)$, $P_1 = (1,0)$, $P_2 = (-1,0)$

$f_{xx} = -4(3x^2-1)$ $\quad f_{xy} = 0$

$f_{yx} = 0$ $\quad f_{yy} = 2$

$$H = \begin{vmatrix} -4(3x^2-1) & 0 \\ 0 & 2 \end{vmatrix}$$

$H(P_0) = 2\cdot(-4)(-1) > 0$ $\quad P_0$ min. loc. forte

$H(P_1) = 2(-4)\cdot 2 < 0$ $\quad P_1$ pto di sella

$H(P_2) = 2(-4)\cdot 2 < 0$ $\quad P_2$ "

a) Piano tangente al grafico di

$$f(x,y) = x^4 + 2xy^3 + 3y^4 - \frac{1}{2}x$$

nel punto $P = (0,0,f(0,0))$.

b) $(0,0)$ è un punto di estremo locale per $f(x,y)$?

a) piano tang in $(x_0, y_0, f(x_0,y_0))$:

$$z - f(x_0,y_0) = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$f_x = 4x^3 + 2y^3 - \frac{1}{2}$$

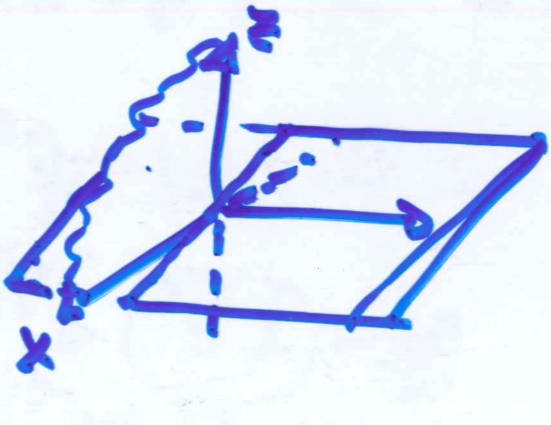
$$f_y = 6xy^2 + 12y^3$$

$$\boxed{f_x(0,0) = -\frac{1}{2} \qquad f_y(0,0) = 0 \qquad f(0,0) = 0}$$

$$z = -\frac{1}{2}x + 0 \cdot y$$

: eq piano tang:

$$\boxed{z = -\frac{1}{2}x}$$



b) NO poiché il piano tang non ha la forma $z = \text{cost}$.