$$\begin{cases} y' = \dfrac{t}{t^2+2} \cdot (y^2+y) \\ y(0) = -2 \end{cases}$$

ED 1° ordine a variabili separabili

$a(t) = \dfrac{t}{t^2+2}$ cont. $\forall t \in \mathbb{R}$

$b(y) = y^2+y$ cont. $\forall y \in \mathbb{R}$

$b'(y) = 2y+1$ " $\forall y \in \mathbb{R}$

$b(y) = 0$: $y(t) = 0$ $y(t) = -1$

nessuna delle 2 è sol. del pr. di Cauchy

separo le variabili:

$$\int \frac{y'\,dt}{y^2+y} = \int \frac{t}{t^2+2}\,dt$$

I.D. $\dfrac{1}{y(y+1)}$: $(-\infty,-1) \cup (-1,0) \cup (0,+\infty)$

$$\int \frac{dy}{y(y+1)} = \int \left(\frac{1}{y} - \frac{1}{y+1}\right) dy = \ln\left|\frac{y}{y+1}\right| + c$$

$$\ln\left|\frac{y}{y+1}\right| = \frac{1}{2}\ln(t^2+2) + c$$

$-2 \in (-\infty,-1)$: la soluz. prende valori in $(-\infty,-1)$

$\ln 2 = \frac{1}{2}\ln 2 + c \Rightarrow c = \frac{1}{2}\ln 2$

$$\ln\left(\frac{y}{y+1}\right) = \frac{1}{2}\ln 2\left(t^2+2\right)$$

$$\frac{y}{y+1} = \sqrt{2\left(t^2+2\right)}$$

$$y = (y+1)\sqrt{2\left(t^2+2\right)}$$

$$y = \frac{\sqrt{2\left(t^2+2\right)}}{1-\sqrt{2\left(t^2+2\right)}}$$

$$\sqrt{2\left(t^2+2\right)} \neq 1$$

$$t^2+2 \neq \frac{1}{2}$$

$$t^2 \neq -\frac{3}{2} \qquad \text{OK !}$$