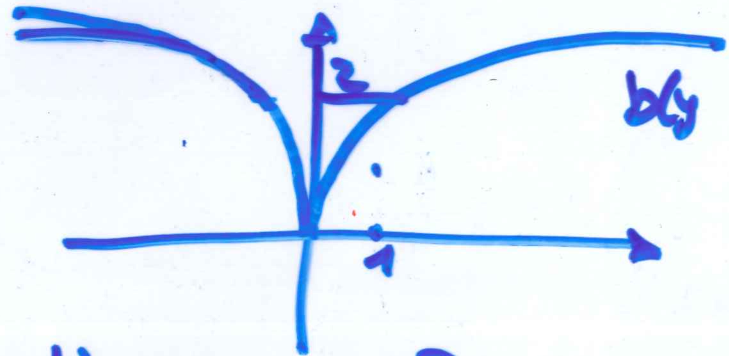
1) $y'(t) = 2\sqrt{|y(t)|}$

1°) ED I ordine variab. separab.

$a(t) = 1 \qquad b(y) = 2\sqrt{|y|}$

a continua su $\mathbb{R}$

b(y)



→ continua su $\mathbb{R}$

$b'(y)$ è definita in $(-\infty, 0) \cup (0, +\infty)$ ed è continua su ciascuno dei 2 intervalli presi separatamente

I probl. di Cauchy hanno sol.

$\forall\, (t_0, y_0) \in \mathbb{R} \times (-\infty, 0)$ o

$(t_0, y_0) \in \mathbb{R} \times (0, +\infty)$

2°) $b(y) = 0 \iff y(t) = 0$: questa è una soluzione part. che corrisponde al problema di Cauchy non evidenziato sopra $y(t_0) = 0, \ \forall t_0 \in \mathbb{R}$

$$\frac{y'}{2\sqrt{|y|}} = 1$$

$$\int \frac{dy}{2\sqrt{|y|}} = \int dt$$

$$\sqrt{|y|} = \begin{cases} \sqrt{y} & \text{se } y>0 \\ \sqrt{-y} & \text{se } y<0 \end{cases}$$

$y>0$: $\displaystyle\int \frac{dy}{2\sqrt{y}} = t + c \qquad c \in \mathbb{R}$

cioè $\sqrt{y} = \underbrace{t+c}_{>0}$ Integr. gen. in forma implicita

$$\begin{cases} y(t) = (t+c)^2 \\ t > -c \end{cases}$$

$y<0$: $\displaystyle -\int \frac{-dy}{2\sqrt{-y}} = t + c \qquad c \in \mathbb{R}$

cioè $-\sqrt{-y} = t+c$

$\sqrt{-y} = -t-c$

$$\begin{cases} -y(t) = (-t-c)^2 \\ -t-c>0 \end{cases} \iff \begin{cases} y(t) = -(t+c)^2 \\ t<-c. \end{cases}$$

$$\begin{cases} y' = 2\sqrt{|y|} \\ y(0) = 3 \end{cases}$$

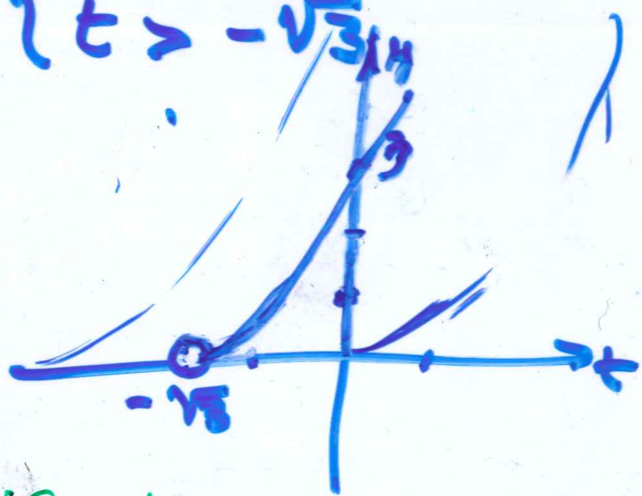
Dato questo problema di Cauchy,

La sol. $y(t)$ deve essere una funz >0

$$\begin{cases} \sqrt{y} = t+c \\ t > -c \\ y(0) = 3 \end{cases} \qquad 0+c = \sqrt{3}$$

La sol diventa
$$\begin{cases} \sqrt{y} = t+\sqrt{3} \\ t > -\sqrt{3} \end{cases}$$

$$\begin{cases} y(t) = (t+\sqrt{3})^2 \\ t > -\sqrt{3} \end{cases}$$



Invece, dato il problema di Cauchy

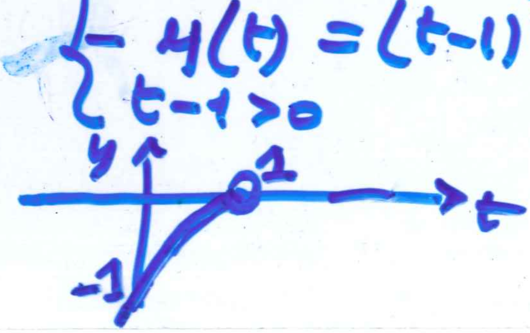
$$\begin{cases} y' = 2\sqrt{|y|} \\ y(0) = -1 \end{cases} \Rightarrow y(t) < 0$$

$$\begin{cases} \sqrt{-y} = -(t+c) \\ y(0) = -1 \end{cases} \qquad -\sqrt{1} = c$$

$$\sqrt{-y} = -t+1$$

$$\begin{cases} -y(t) = (t-1)^2 \\ t-1 > 0 \end{cases}$$

$$\begin{cases} y(t) = -(t-1)^2 \\ t < 1 \end{cases}$$



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2) $y' - \frac{2}{x^2} y = \frac{1}{x^4}$

1°) EDL 1° ordine completa

2°) omog assoc. $z' = \frac{2}{x^2} z$

$z = 0$ è sol.

$$\int \frac{dz}{z} = \int \frac{2dx}{x^2}$$

$$\ln|z| = -\frac{2}{x} + c$$

$$z = \underbrace{\pm e^c}\, e^{-2/x}$$

$$z(x) = k\, e^{-2/x} \qquad k \in \mathbb{R}$$

3°) Sol part. (VAR. COSTANTI)

$$\bar{y}(x) = k(x)\, e^{-2/x}$$

$$\bar{y}' = k' e^{-2/x} + k\, e^{-2/x} \cdot \frac{2}{x^2}$$

Sostituisco

$$e^{-2/x}\left(k' + k \cdot \cancel{\frac{2}{x^2}} - \cancel{\frac{2}{x^2}} k\right) = \frac{1}{x^4}$$

$$k' = \frac{1}{x^4} e^{2/x}$$

$$\int \frac{1}{x^4} e^{2/x} dx = \qquad \int -\frac{1}{2} \cdot \frac{t^2}{4} e^t dt \underset{\text{PP}}{=}$$

FD $e^t dt$

$\frac{2}{x} = t$

$\frac{1}{x} = \frac{t}{2}$ $\qquad -\frac{2}{x^2} dx = dt$

$$= -\frac{1}{8}\left(t^2e^t - \int 2te^t\,dt\right) =$$

$$= -\frac{1}{8}\left(t^2e^t - 2te^t + \int 2e^t\,dt\right) =$$

$$= -\frac{1}{8}\left(t^2 - 2t + 2\right)e^t + c =$$

$$= -\frac{1}{2}\left(\frac{4}{x^2} - \frac{4}{x} + 2\right)e^{2/x} + c = k(x)$$

$$c = 0$$

$$\bar{y}(x) = -\frac{1}{2}\left(\frac{4}{x^2} - \frac{4}{x} + 2\right)\underbrace{e^{2/x}\cdot e^{-2/x}}_{=1} =$$

$$= -\frac{1}{2}\left(\frac{4}{x^2} - \frac{4}{x} + 2\right) =$$

$$= -\frac{2}{x^2} + \frac{2}{x} - 1$$

$$y(x) = -\frac{2}{x^2} + \frac{2}{x} - 1 + ke^{-2/x}.$$