GSD 2017 – titles & abstracts

Final version

F. Calogero* *(Roma)*

Zeros of polynomials and solvable nonlinear evolution equations

`Abstract:` Recent findings concerning the relations between the coefficients and the zeros of time-dependent monic polynomials will be reported. They underline a differential algorithm to compute all the zeros of a generic polynomial and allow the identification of novel classes of solvable nonlinear evolution equations, including many-body problems and nonlinear evolution partial differential equations, as well as endless hierarchies of such solvable models. Part of this work has been done with Oksana Bihun, with Mario Bruschi and with Francois Leyvraz.

C. Chanu *(Torino)*

Extended Hamiltonian systems, classical and quantum aspects

`Abstract:` In recent years, many natural Hamiltonian systems, classical and quantum, with constants of motion of high degree, or symmetry operators of high order, have been found and studied. Most of these Hamiltonians, in the classical case, can be included in the family of Extended Hamiltonians, geometrically characterized by the structure of Warped Manifold of their configuration manifold. For the extended manifolds, the characteristic constants of motion of high degree are polynomial in the momenta of a recursively determined form. After a brief survey recalling the origin of the notion of extended Hamiltonian, some recent developments forward quantum extended Hamiltonian operators with analogous properties will be presented.

A. Degasperis *(Roma)*

Wave instabilities in integrable systems

`Abstract:` The Benjamin-Feir instability of periodic waves, which occurs in water dynamics and optics, is investigated for periodic waves which are modelled by two coupled Nonlinear Schroedinger equations. Because of the coupling the linear (short time) instability depends on several parameters. This makes unstable behavior far richer than that described by a single Nonlinear Schroedinger equation where the main mechanism is due to self-focusing. To illustrate a variety of processes we consider the stability properties of two coupled continuous waves with different wave numbers, amplitudes and coupling constants. In contrast to the standard approach which relies on Fourier analysis, our method makes use of the integrability of the wave propagation model, and of the spectral properties of the Lax pair. This method can be generalized so as to be applicable to quite a large number of integrable wave models.

G. Derks *(Guildford)*

Dimer Dynamics and Degenerate Transversally Intersecting Manifolds

`Abstract:` We consider a pharmacological model of dimerisation, i.e., a receptor binding two ligand (drug) molecules. This model is an extension of the well studied target mediated drug disposition model (TMDD) in which the receptor binds to one ligand molecule. It is assumed that the binding is the fastest process. This gives a separation of time scales, which allows us to use geometric singular perturbation theory to analyse these models. In both models, the slow manifold consists of two components, which intersect transversely in the origin. The dimerisation model leads to a degenerate intersection. To analyse such intersection, we consider a general two parameter slow-fast system in which the critical set consists of a one dimensional manifold and a two dimensional manifold, intersecting at the origin. Using geometric desingularisation, we determine the fate of the incoming one dimensional manifold and show that for a subset of the parameter set there is a jump away from the intersection at the origin and away from the critical set. For the remaining parameter set, there is an exchange of stability between the attracting components of the critical set and the direction of the continuation can be expressed in terms of the parameters. The parameters for the dimerisation model fit into the latter category and we will give the approximation of the dynamics in the dimerisation model. This is joint work with Philip Aston and Chrissy Gavin.

M. Dunajski *(Cambridge)*

Metrisability of Painlevé equations

`Abstract:` not available

G. Gaeta *(Milano)*

Unusual geometry and symmetry in liquid crystals

`Abstract:` Recent works considered liquid crystals (or other materials, see https://www.flickr.com/photos/nonlin/4994885462/) described – at the level of Landau theory – by an order parameter in the form of a third order tensor. One has then to minimize a cubic potential constrained to the unit sphere. Despite the simplicity of such a problem (just one step beyond minimizing a constrained quadratic form!) the detailed analysis displays rather unusual geometry and symmetry properties. The talk will suffer from the absence of a beamer, is related to the problem to be discussed by S. Walcher, and is based on joint work with E. Virga [DOI 10.1140/epje/i2016-16113-7]. (See also https://arxiv.org/abs/1503.04595 and https://arxiv.org/abs/1701.06761)

A. Giacobbe *(Catania)*

Quasi periodicity of dynamical systems with symmetries

`Abstract:` Assume to be given a dynamical system on a manifold M invariant under the action of a Lie group G. Under which conditions the quasi-periodicity of the reduced system implies the quasi-periodicity of the original system? Does these condition have any relation with the existence of an appropriate connections of the principal bundle (M,M/G)?

H. Hanßmann *(Utrecht)*

Hamiltonian oscillators in $1{:}\pm 2{:}\pm 4$ **resonance**

`Abstract:` The 1:2:4 resonance is one of the four definite resonances of genuinely first order and thus known to be non-integrable. The frequency ratios provide unfolding parameters (but note that the dynamic phenomena can also occur in a single system of 6 or more degrees of freedom). The indefinite versions of the resonance do not require the equilibrium to be a local extremum of the Hamiltonian. Normalization yields a normal form approximation and the resulting (non-integrable) system can be reduced to 2 degrees of freedom.
The non-trivial isotropies of the two coupled $1{:}\pm 2$ resonances prevent the reduced phase space from being a smooth manifold but the dynamics on the singular part is in fact easier to understand. On the regular part of the reduced phase space the distribution of equilibria turns out to be determined by a single polynomial of degree 4. These are the relative equilibria that determine the behaviour of the 3 normal modes when passing through the resonance.

A. Marchesiello *(Prague)*

Superintegrable 3D systems in a magnetic field and Cartesian separation of variables

`Abstract:` We consider three dimensional superintegrable systems in a magnetic field. We study the class of such systems which separate in Cartesian coordinates in the limit when the magnetic field vanishes, i.e. possess two second order integrals of motion of the "Cartesian type". For such systems we look for additional integrals up to second order in momenta which make these systems minimally or maximally superintegrable and study the corresponding trajectories. We observe that the leading structure terms of the Cartesian type integrals should be considered in a more general form than for the case without magnetic field

F. Oliveri *(Messina)*

Decoupling of general quasilinear first order systems in two independent variables

`Abstract:` The decoupling problem of general quasilinear first order systems involving two independent variables is considered. We face either the case of homogeneous and autonomous systems or the one of nonhomogeneous and/or nonautonomous systems. Necessary and sufficient conditions for the partial or full decoupling of the systems at hand are provided. The conditions involve the properties of eigenvalues and eigenvectors of the coefficient matrix, and provide the differential constraints whose integration leads to the decoupling transformation. Some applications of physical interest are also given. [See:
M. Gorgone, F. Oliveri, M. P. Speciale, On the decoupling problem of general quasilinear first order systems in two independent variables, *J. Math. Anal. Appl.* **446**, 276–298, 2017.]

G. Panati *(Roma)*

Symmetry, localization and topology in solid state physics: The Localization Dichotomy for gapped quantum systems

`Abstract:` not available

C. Pantazi *(Barcelona)*

Non integrability of planar polynomial systems

`Abstract:` We study a necessary condition for the integrability of polynomial vector fields in the plane using a similar result to the one of Morales, Ramis and Simó on Hamiltonian systems. We provided a necessary condition for the existence of a meromorphic first integral using the variational equations around an invariant curve. We illustrate this theorem with several families of examples. A key point in these applications is to check wether a suitable primitive is or not elementary. Using a theorem by Liouville, the problem is equivalent to the existence of a rational solution of a certain first order linear equation, the Risch equation. This is a classical problem studied by Risch in 1969, and the solution is given by the "Risch algorithm". This is a joint work with Primitivo B. Acosta-Humnez (Universidad Simón Bolivar, Colombia), J. Tomas Lázaro (Universidad Politécnica de Barcelona, Spain) and J. Morales-Ruiz (Universidad Politécnica de Madrid)

M. Pavlov *(Moscow)*

Integrable Systems, the High-Frequency Limit

`Abstract:` In this talk we consider a special class of integrable systems, which possess the so called high-frequency limit. We construct point transformations connecting these integrable systems with their high-frequency limits. This property allows to construct new types of solutions.

M. Procesi* *(Roma)*

Stability and instability for the NLS equation on $\mathbf{T}^2$

`Abstract:` I will discuss some results on the existence and stability of quasi-periodic solutions and on the transfer of energy from low to high Fourier modes for the Nonlinear Schrodinger equation on the two dimensional torus.

G. Pucacco *(Roma)*

Divergent Series: a smart use of Perturbation Theory

`Abstract:` The predictive ability of perturbation theories based on Hamiltonian normal forms can be extended by suitable algorithms of summation of divergent series. The predictions are often reliable even outside the convergence radius of the original Hamiltonian and we implement these results by using re-summation techniques of asymptotic series. We apply these methods to several problems in mechanics and astrodynamics by looking for analytical expressions of invariant objects in phase-space.

P. Santini *(Roma)*

Analytic description of the exact rogue wave recurrence in the periodic NLS Cauchy problem

`Abstract:` Rogue Waves (RWs) are transient waves appearing, apparently from nowhere, in several physical contexts, like in water waves, nonlinear optics, Bose-Einstein condensates, liquid Helium, etc ..[1]. It is by now well understood that the basic physical mechanism for the appearance of RWs is the Modulation Instability of the amplitude of quasi monochromatic waves [2],[3], and the simplest nonlinear model for the description of such phenomenon is the integrable [4] self-focusing Nonlinear Schroedinger Equation (sfNLS) and its unstable background, described by the simple x-indepenedent exact solution u=exp(2 i t). We have recently solved the Cauchy problem of the sfNLS equation for generic, x-periodic, initial perturbations of the background, in the case of N unstable modes. The solution of the problem, given in terms of elementary functions in different time intervals, describes the exact recurrence of an infinite sequence of rogue waves, and the n-th rogue wave of the sequence is described, in the finite time interval in which it appears, by the N-breather solution of Akhmediev type, whose paremeters are expressed in terms of the initial data via elementary functions. The above results have been obtained in [5] and in [6] through two different approaches: the finite gap method and, respectively, matched asymptotic expansions techniques. [This is joint work with P. G. Grinevich.]
References: [1] M. Onorato et al., *Phys. Rep.* 528 (2013) 47-89. [2] T. B. Benjamin, J. E. Feir, *J. Fluid Mech.* 27 (1967) 417-430. [3] V. E. Zakharov, *J. Appl. Mech. Techn. Phys.* 9 (1968) 190-194. [4] V. E. Zakharov and A. B. Shabat, *Sov. Phys. JETP* 34, 1, 62-69, 1972. [5] P. G. Grinevich and P. M. Santini, Analytic description of the exact recurrence of rogue waves in the periodic NLS setting, using the finite gap method. Preprint (2017). [6] P. G. Grinevich and P. M. Santini, Analytic description of the exact recurrence of rogue waves in the periodic NLS setting, using matched asymptotic expansions. Preprint (2017).

F. Verhulst[1] *(Utrecht)*

The FPU chain with alternating masses

`Abstract:` The periodic Fermi-Pasta-Ulam problem with alternating masses presents complicated dynamics. In the case of $2n$ particles the use of symmetries in the analysis generalizes to three exact periodic solutions and families of invariant manifolds (bushes). We analyze the cases of four and eight particles to find periodic solutions and invariant manifolds, but with a dynamics that is strongly affected by the choice of the alternating mass m. Normal form calculations help to identify quasi-trapping regions leading to delay of recurrence. The results suggest that equipartition of energy near stable equilibrium is not probable.

[1]Ferdinand also agreed, as required by other participants, to give a second talk. This will be on "Recurrence and the fine-structure of resonance zones"

R. Vitolo* *(Lecce)*

New results on Dubrovin-Novikov Hamiltonian operators and integrable systems

`Abstract:` Dubrovin-Novikov Hamiltonian operators have interesting geometric properties. First-order operators have been systematically investigated. The geometry of third-order operators is a recent discovery. We review recent results that associate integrable systems of PDEs with third-order operators.

S. Walcher *(Aachen)*

Other peoples' work on eigenvectors of tensors

`Abstract:` The notion of eigenvector of a linear map admits a natural generalization to "eigenvectors" of homogeneous polynomial maps of degree 2 or greater. (The notion of eigenvalue is not well-defined and much less relevant in this context.) Existence problems and results about the number of eigenvectors can be discussed with the help of Bezout's Theorem on projective space. Some results and applications are given.