Serie numeriche (L. V., 27-04-01)

Studiare la convergenza assoluta e/o semplice della serie data.

1. $\displaystyle\sum_{n=1}^{\infty} \log\left(\frac{n^3+2}{n^2+3}\right)$ **2.** $\displaystyle\sum_{n=2}^{+\infty} \big(|a|-1\big)^n$ **3.** $\displaystyle\sum_{n=7}^{+\infty} \big(\sqrt{n+1}-\sqrt{n}\big)^2$

4. $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{\sqrt{n}+3}{2n+5}\right)^a$ **5.** $\displaystyle\sum_{n=2}^{+\infty} \left(\frac{\sqrt{a}+3}{2a+5}\right)^n$

6. $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n}{n\sqrt{3+\log n}}$ **7.** $\displaystyle\sum_{n=1}^{+\infty} \big|\log\big(1-(n-2)^{-2}\big)\big|^a$

8. $\displaystyle\sum_{n=2}^{+\infty} \frac{(-1)^n}{\log n}+\left(\frac{\sin n}{n}\right)^2$ **9.** $\displaystyle\sum_{n=1}^{+\infty} \frac{2(\sin n)^2-1}{3n^3+(-1)^n}$

10. $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{6}{10-2\arctan n}\right)^n$ **11.** $\displaystyle\sum_{n=1}^{+\infty} \left(n\sin\frac{1}{n}\right)^{n^3}$

12. $\displaystyle\sum_{n=1}^{+\infty} \left(\sqrt[n]{n^3+4n}-1\right)\frac{n}{1+\log n}$ **13.** $\displaystyle\sum_{n=1}^{+\infty} \big(\sqrt{n(n+1)}-\sqrt{n(n-1)}-1\big)$

14. $\displaystyle\sum_{n=1}^{+\infty} \frac{n^n n!}{(2n)!}$ **15.** $\displaystyle\sum_{n=1}^{+\infty} a^{n!} \quad (a\in\mathbf{R})$

16. $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{\sqrt[n]{\log n}}$ **17.** $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{n-1}{n+1}\right)^{n(n-1)}$

18. $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n^{1+(1/n)}}$ **19.** $\displaystyle\sum_{n=1}^{+\infty} \left(n^{n^a}-1\right) \quad (a\in\mathbf{R})$

20. $\displaystyle\sum_{n=1}^{+\infty} (-1)^n\left(\frac{2n+1}{2n}\right)$ **21.** $\displaystyle\sum_{n=1}^{+\infty} (-1)^n\frac{\sqrt{n}}{n+100}$

22. $\displaystyle\sum_{n=4}^{+\infty} \frac{\sqrt{n}}{\big(1+e^n\big)^a+n^b\log n} \quad (a,b>0)$ **23.** $\displaystyle\sum_{n=1}^{+\infty} (-1)^{\frac{n(n-1)}{2}}2^{-n}$

24. Calcolare la somma della serie $\displaystyle\sum_{n=1}^{+\infty}(a_n-a_{n+2})$ dove $\{a_n\}$ è una successione convergente a 0.

25. a) Trovare due successioni $\{a_n\}, \{b_n\}$ di numeri reali tali che le serie $\sum a_n$, $\sum b_n^2$ siano convergenti mentre le serie $\sum a_n^2$, $\sum b_n$ non lo siano.
b) Dimostrare che, se $c_n \geq 0$ definitivamente e la serie $\sum c_n$ converge, allora anche $\sum c_n^2$ converge.