

3 1 1 1 5
T C

EPIMORFISMI REGOLARI

U

split $epi \Rightarrow reg\ epi \Rightarrow epi$
" " $\not\Leftarrow$ $\not\Leftarrow$
retrazione $\not\Leftarrow$

Top $epi\ regulari : f: X \rightarrow Y$

survelli e con Y dotato
della top generata risp a f

EX Dim che e nel caso $f \equiv$
 $Coep(h, k)$ con $h, k: Z \rightarrow X$

Grp $e \in \underline{AB}$

regular $epi = epi =$
survelli

G: Mod \rightarrow Set

G dimenticate come su Set

$F \vdash G$

F : costruzione di Q
libere

$X \in \text{Set}$

$\mathbb{F}X =$ monoida libera su Set

$\mathbb{G}\mathbb{F}X =$ insieme delle sequenze finite (stringhe) di elt: $\in X$ compresa quella vuota $()$

$(x_1, x_2, \dots, x_n) \in \mathbb{G}\mathbb{F}X$

multiplicazione e $\mathbb{G}\mathbb{F}X$ edotato concatenazione

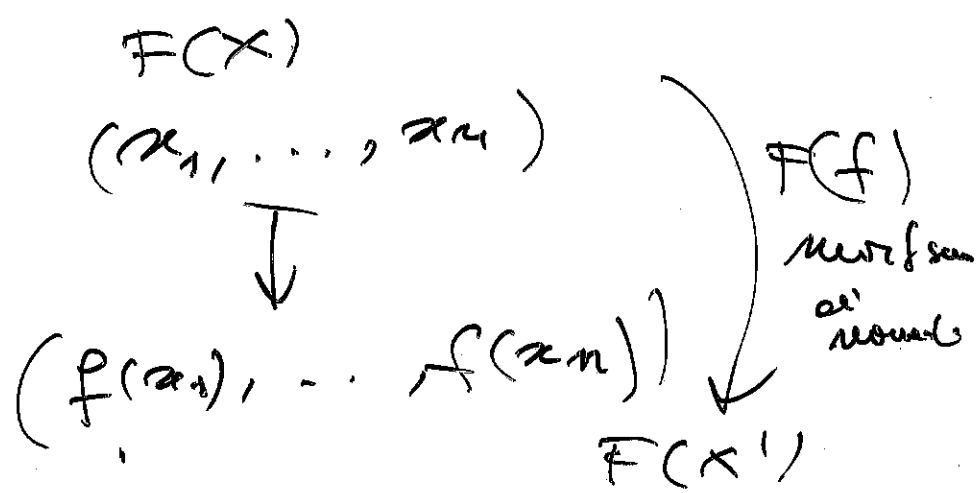
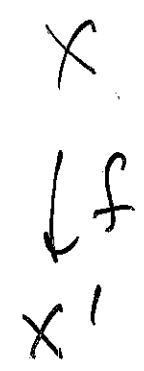
$(x_1, \dots, x_n) \cdot (y_1, \dots, y_m) =$

$(x_1, \dots, x_n, y_1, \dots, y_m)$

$() \cdot (x_1, \dots, x_n) = (x_1, \dots, x_n) \cdot () =$

(x_1, \dots, x_n)

luego un monoida $\mathbb{F}X$



$$\Sigma_{FX} : FGFX \longrightarrow FX \quad (6)$$

$$G(\Sigma_{FX}) : \underbrace{GF}_{T^2 X} GF X \longrightarrow \underbrace{GF X}_{TX}$$

$$\mu_X : ((x_1, x_2), (x_3)) \longmapsto (x_1, x_2, x_3)$$

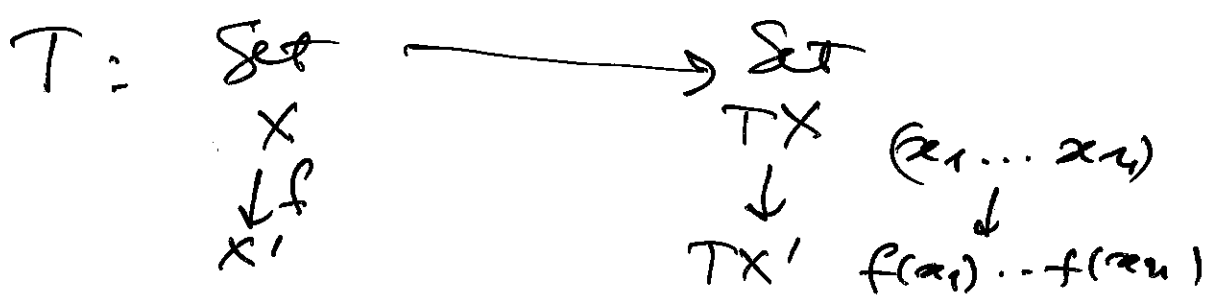
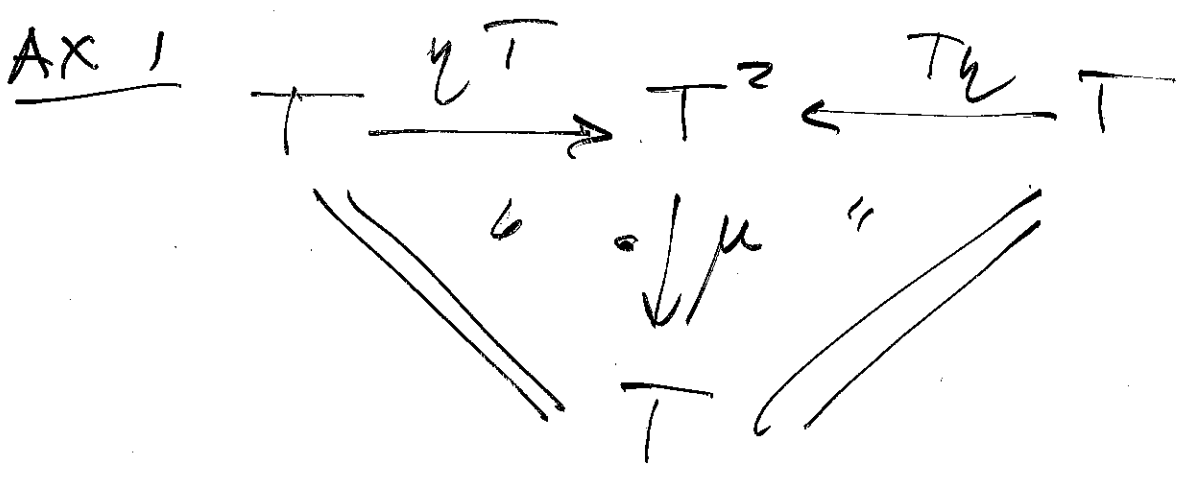
de loops de $\mu : T^2 \longrightarrow T$

~~Esta parte precisa~~ $\mu = \sigma \circ \Sigma F$ é uma t.u

(T, η, μ) de loops a uma

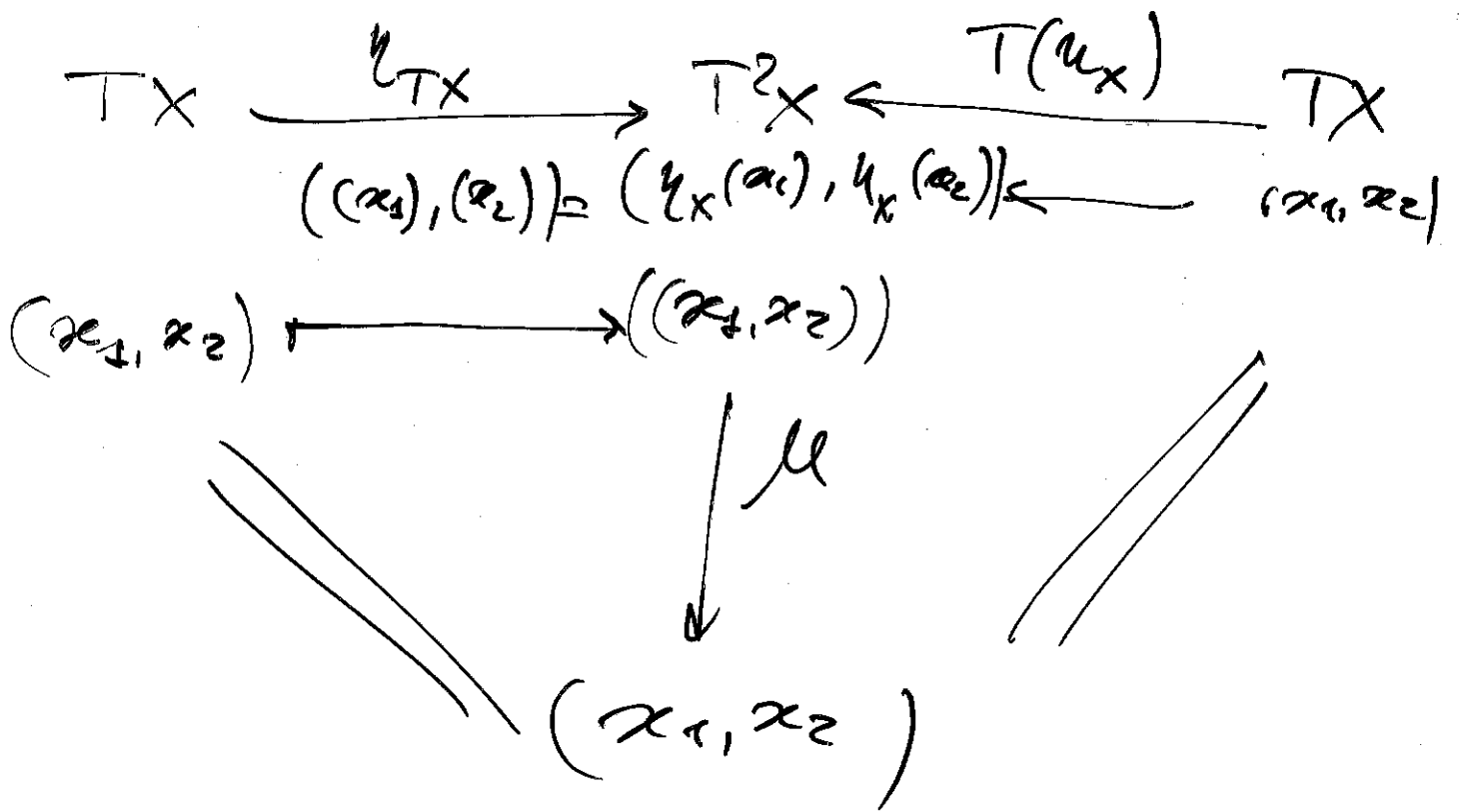
monoidal (see Set) $T : \underline{\text{Set}} \longrightarrow \underline{\text{Set}}$

se provamos que valemos:

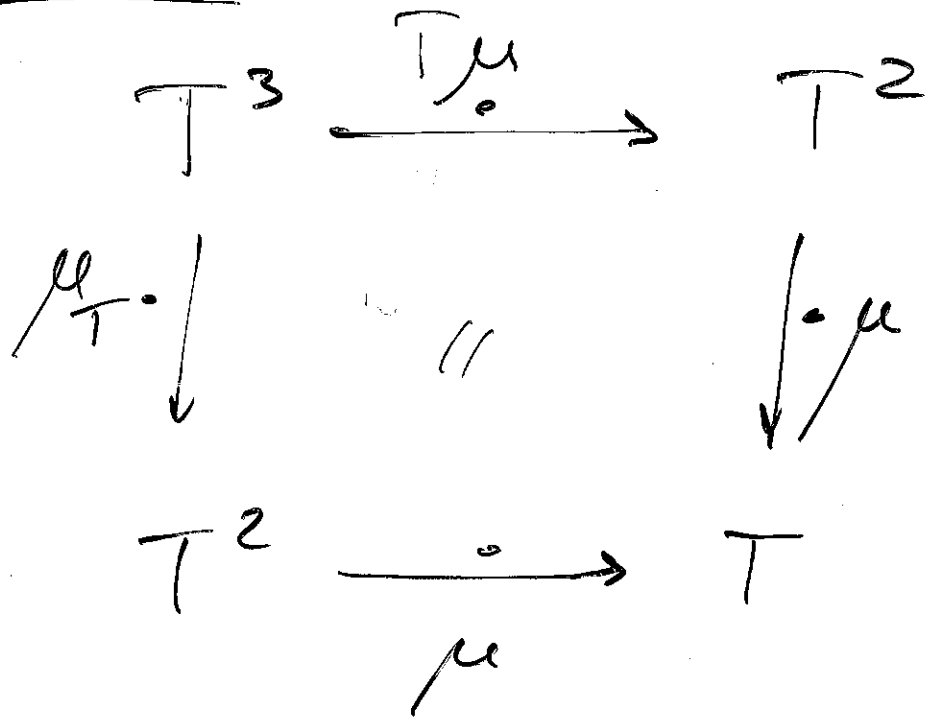


case $\forall x \in \underline{\text{Set}}$

(5)



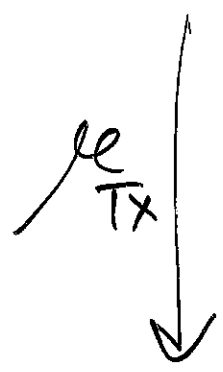
2 AX



$T^3 X$
 ψ

$$((x, y), (z)) \xrightarrow{T(\mu)} (\mu_x((x, y)), \mu_x((z)))$$

$((x, y), (z))$



$$(x, y, z) \xrightarrow{\mu} (x, y, z)$$

Df Une monade sur $\mathbb{C} \mathbb{R}^1$
dotée de (T, η, μ) above

$$T : \mathbb{C} \longrightarrow \mathbb{C}$$

$$\eta : \text{Id}_{\mathbb{C}} \longrightarrow T$$

$$\mu : T^2 \longrightarrow T$$

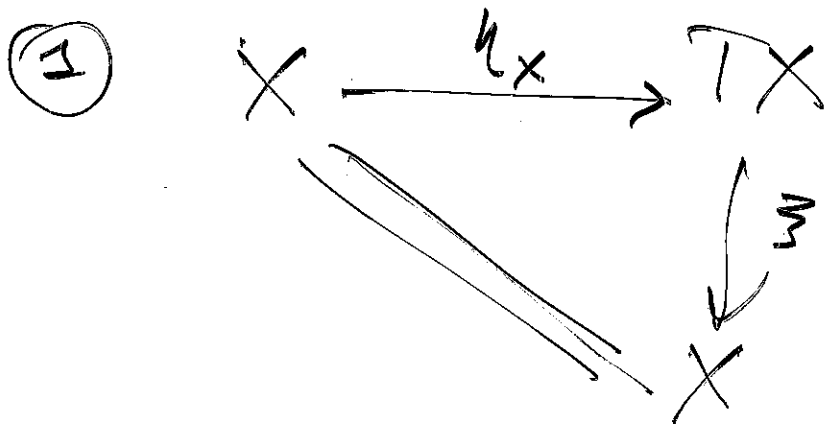
avec $A \times \mathbb{R} \cong \mathbb{R}^2$

Def Un'algebra per T su \mathcal{C} è una \mathcal{C}

coppia (X, \mathbb{T}) $X \in \text{ob } \mathcal{C}$

$\mathbb{T}: TX \longrightarrow X$ in \mathcal{C} detta
 morfismo strutturale t.c.

sono verificati gli assiomi



nel caso dei monoidi

$$\mathbb{T}: TX = GX \longrightarrow X$$

$$(x, y) \longmapsto \mathbb{T}(x, y) =: x \cdot y$$

$$() \longmapsto \mathbb{T}() =: 1$$

$$(x) \longmapsto \mathbb{T}(x) = x$$

$$x \longrightarrow (x)$$

$$\downarrow \mathbb{T}$$

$$x$$

$$\begin{array}{ccc}
 \underline{A \times Z} & T^2 X & \xrightarrow{T(\cong)} TX \\
 \mu_X \downarrow & = & \downarrow \cong \\
 TX & \xrightarrow{\cong} & X
 \end{array}$$

nel caso di monoidi

$$\begin{array}{ccc}
 (a), (b, c) & \xrightarrow{\quad} & (a, b \cdot c) \\
 \mu \downarrow & T(\cong) \downarrow & \\
 (a, b), c & \xrightarrow{\quad} & (\cong(a, b), \cong(c)) \\
 & & \text{"} \\
 & & (a \cdot b, c) \\
 \mu \downarrow & & \downarrow \cong \\
 (a, b, c) & \xrightarrow{\quad} & \cong(a, b, c) = (a \cdot b) \cdot c \\
 & & \text{"} \\
 & & a \cdot (b \cdot c)
 \end{array}$$

\Rightarrow $\cong(a, b)$ definese un prodotto associativo

$$\left(\begin{matrix} (0), (a) \\ (a), () \end{matrix} \right) \xrightarrow{T(\mathbb{Z})} \left(\begin{matrix} \mathbb{M}(a), \mathbb{M}() \\ \mathbb{P}^s, \mathbb{H}^s \end{matrix} \right) \quad (9)$$

$$\begin{array}{ccc} \downarrow \mu & & \downarrow \mathbb{M} \\ (a) & \xrightarrow{\quad} & \mathbb{M}(a) = a \circ f \\ & \searrow \mathbb{M} & \uparrow a \end{array}$$

(X, \cdot) è un monoidale

Def Un morfismo tra 2 T -algebra
(\mathcal{A} algebra per la monoidale T)

$$f: (X, \mathbb{M}) \longrightarrow (X', \mathbb{M}') \quad \bar{f}$$

è un morfismo $f: X \longrightarrow X'$ t.c.

$$\begin{array}{ccc} TX & \xrightarrow{Tf} & TX' \\ \mathbb{M} \downarrow & = & \downarrow \mathbb{M}' \\ X & \xrightarrow{f} & X' \end{array}$$

per i monoidi:

(10)

$$(a, b) \longrightarrow (f(a), f(b))$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ a \cdot b & \xrightarrow{f} & f(a \cdot b) = f(a) \cdot f(b) \end{array}$$

de loops e un morfismo di monoidi

corrispondente è la stessa che è \mathbb{C}

con almeno costante una

$$\text{categoria } \mathbb{C}^T = \text{Alg}_T(\mathbb{C})$$

della T -algebra

$$(TX, \mu_x) \quad \mu_x : T(TX) \longrightarrow TX$$

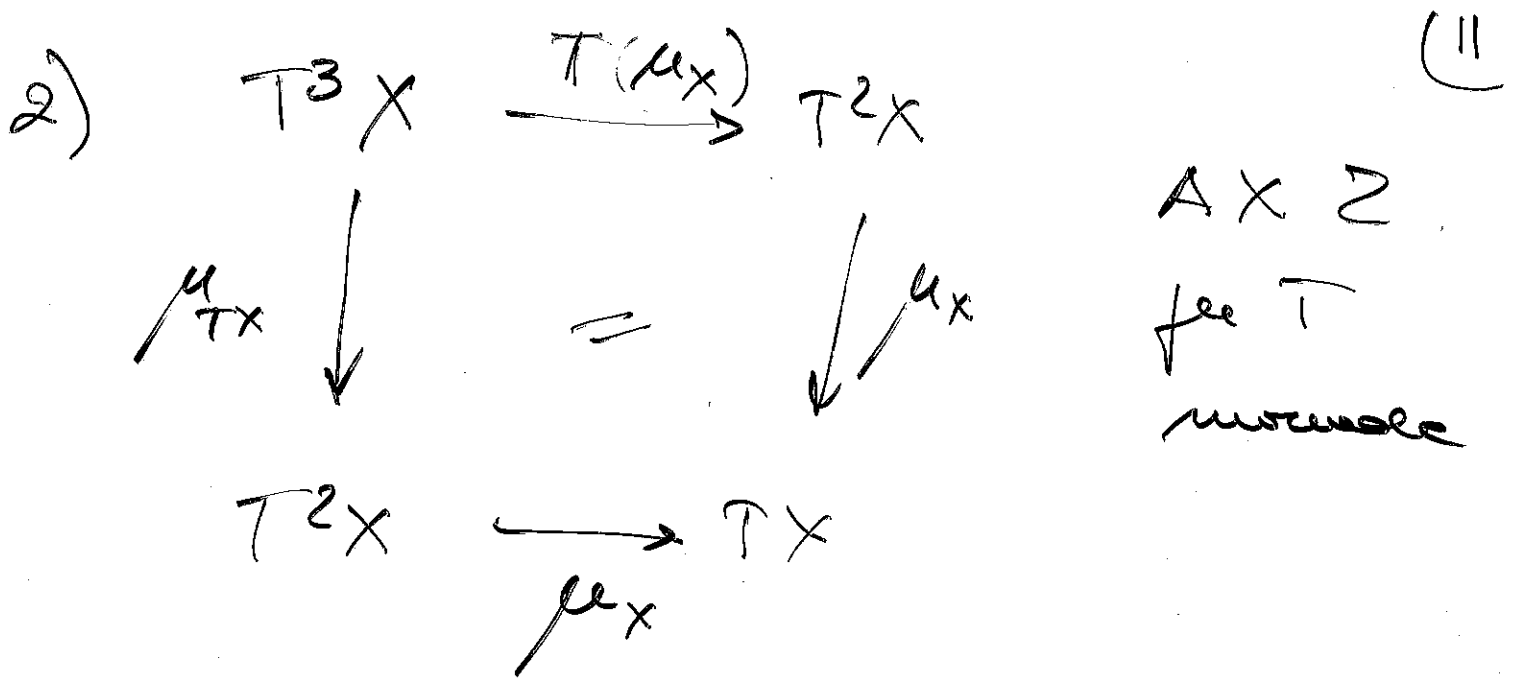
è una T -algebra

$$\forall X \in \text{ob}(\mathbb{C})$$

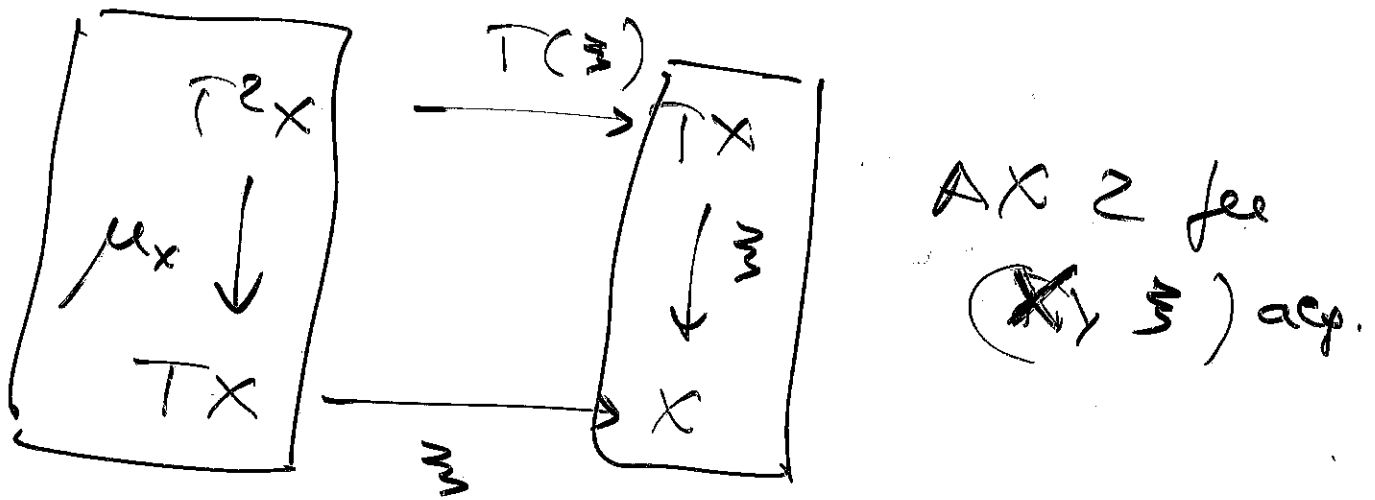
(algebra libera)

$$\begin{array}{ccc} TX & \xrightarrow{\eta_{TX}} & T^2X \\ & \searrow & \downarrow \mu_x \\ & & TX \end{array}$$

I parte
dell'AXI
per T monoidi



now solve!



$$\mathbb{3} : (TX, \mu_X) \longrightarrow (\cancel{TX}, (X, m))$$

\bar{e} se riferisce al'assoc. mc

EX 1) Cos'è una monoidale se \underline{X} , for X monoidale?

$$2) \quad \text{Ab} \begin{array}{c} \xrightarrow{J} \\ \xleftarrow{F} \\ \xrightarrow{F} \end{array} \text{Grp} \quad F \dashv J$$

(i) mostrare che $JF: \text{Grp} \rightarrow \text{Grp}$ è una monade su Grp

(i') trovare le algebre per questo monade

$$3) \quad U: \underline{\text{Poset}} \longrightarrow \underline{\text{Set}}$$

↑
riservati

pos. ordinati e funzione monotone

(i) mostrare che U è oggetto di

$$\exists F \dashv U$$

(i') mostrare che $UF: \underline{\text{Set}} \rightarrow \underline{\text{Set}}$ monade su Set

(i'') trovare le algebre per UF

$$4) \quad P: \underline{\text{Set}} \longrightarrow \underline{\text{Set}}$$

$$\begin{array}{ccccc} X & \longrightarrow & P(X) & \ni & A \\ \downarrow P & & \downarrow P(S) & & \downarrow \\ Y & & P(Y) & \ni & f(A) \end{array}$$

$$\begin{array}{ccc} \uparrow & x & \longrightarrow \mathbb{P}(x) \\ \cup_x & x & \longrightarrow \{x\} \end{array}$$

ε t.h.

$$\begin{array}{ccc} \mu_x : \mathbb{P}(\mathbb{P}(x)) & \longrightarrow & \mathbb{P}(x) \\ \uparrow \cup & & \\ x & \longrightarrow & \cup x \end{array}$$

μ e t.h.

(i) mostrare che abbiamo una monade

(ii) trovare le sue algebre.