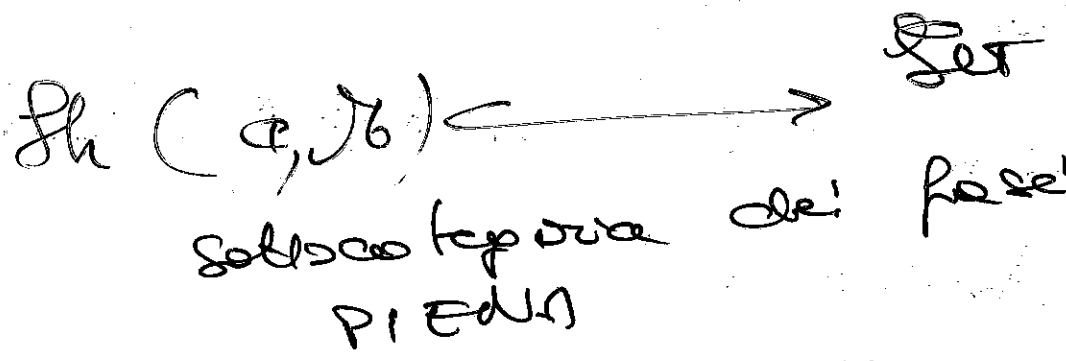


4/12/15

TC

Df Un topò e' Grothendieck
 è una categoria equivalente
 a $Sh(\mathcal{C}, \mathcal{J})$ $(\mathcal{C}, \mathcal{J})$ sub
 \mathcal{C}^{op}



è dimostrato essere una localizza
 zone del topò Set

$\Rightarrow Sh(\mathcal{C}, \mathcal{J})$ è un topò
 (sotto topò di Set)

Topò di Gr. \Rightarrow topò (elementare)
 \uparrow

hanno
 cofinità
 infinite

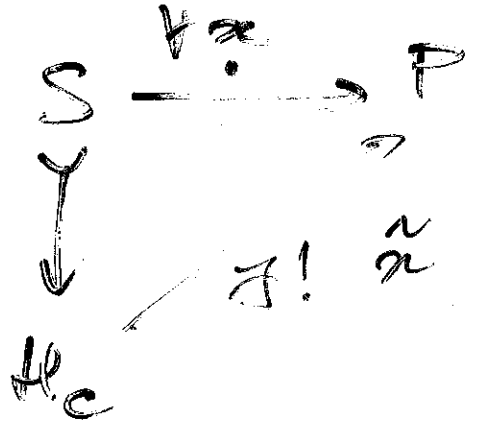
Set fin
 \uparrow
 no

THE GIRAUD : CARATTERIZZAZIONE
 TOPÒS di GRUPP.

ESERCIZI

1. \mathcal{C} piccola
 $\forall C \in \mathcal{C} \quad \mathcal{I}_C(C) = \{H_C\}$
 è una topologia (la + piccola)

$\mathcal{B} \in \text{Set}$ σ of σ un insieme \mathcal{C}
 di $\forall C \quad \forall S \in \mathcal{I}_C(C)$

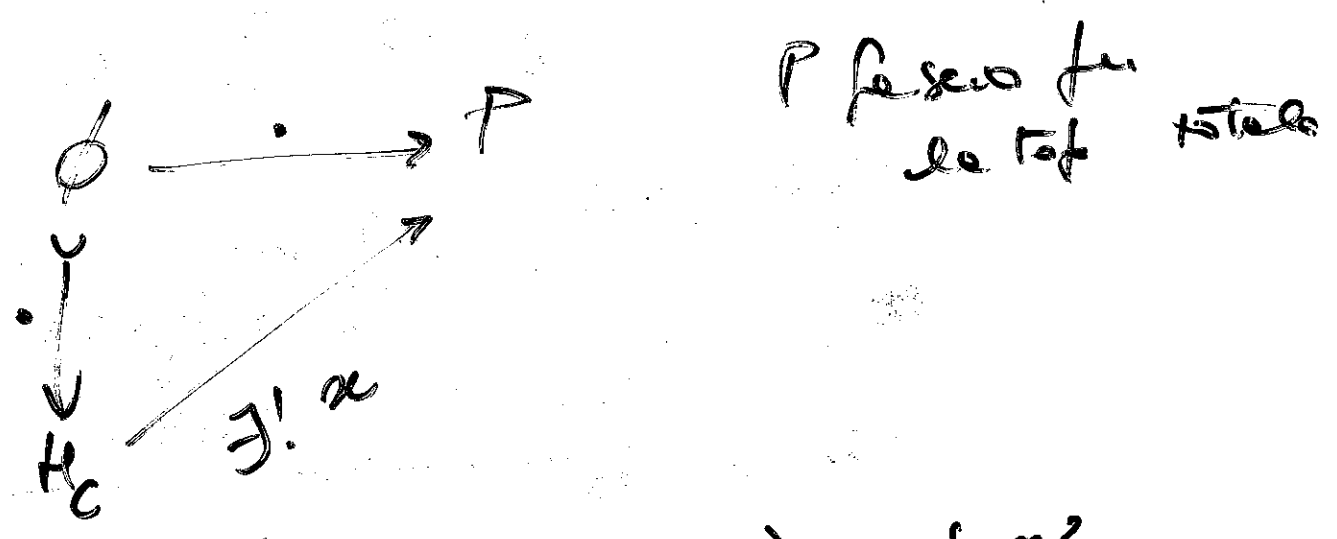
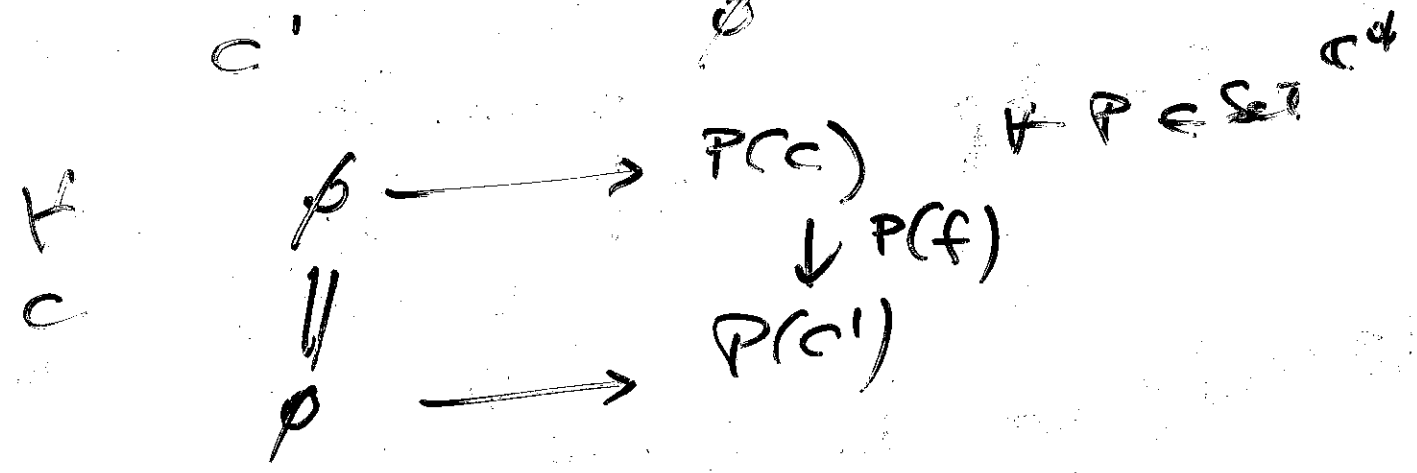
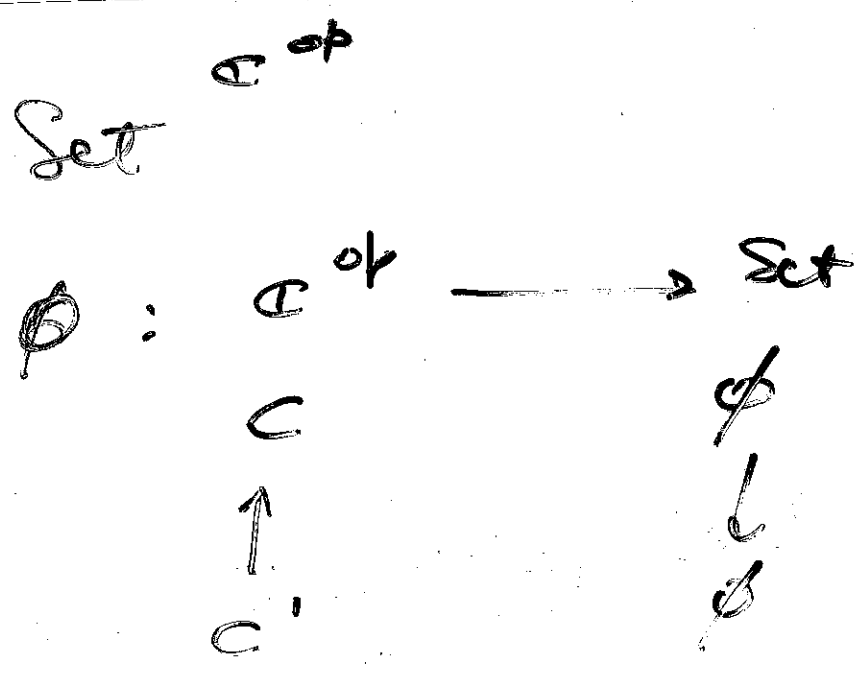


$\mathcal{C} \Rightarrow H_C$ ogni funzione σ \Rightarrow σ \Rightarrow σ
 fanno per questa topologia (bassa)
 quindi $\text{Set} = \mathcal{I}(\mathcal{C}, \mathcal{I})$ \Rightarrow
 per top σ \Rightarrow σ .

2. \mathcal{C} \Rightarrow \mathcal{C} \Rightarrow \mathcal{C}

$S \in \mathcal{I}_C(C) \Leftrightarrow S \supseteq H_C$ \Rightarrow
 una collezione \mathcal{C}
 \Rightarrow \mathcal{C} \Rightarrow \mathcal{C}

\Rightarrow \mathcal{C} \Rightarrow \mathcal{C}
 \Rightarrow \mathcal{C} \Rightarrow \mathcal{C}



th.u. $(\mathcal{H}_c, \mathcal{P}) = \{*\}$

How $\text{Set}^{\mathcal{C}^{op}}$

||S Y

$\mathcal{P}(\mathcal{C})$

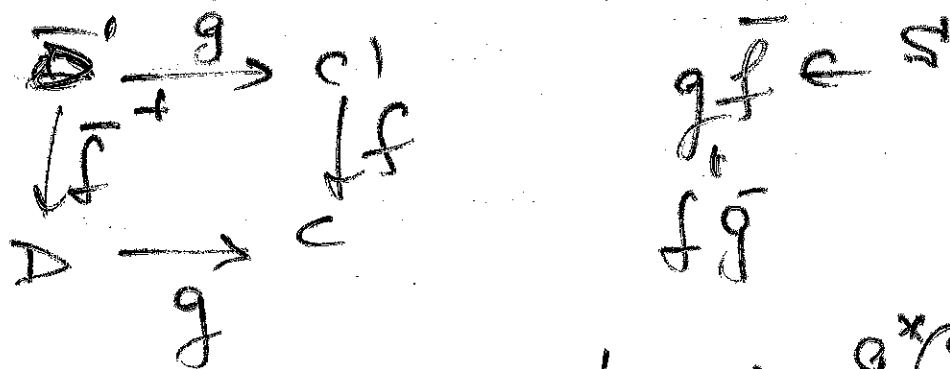
$\Rightarrow \mathcal{P} \bar{c}$ constante

3. \mathbb{R} in funzione

$S \in \mathcal{C}(a) \Rightarrow S \neq \emptyset$ Sussidiario

4. \mathbb{R}

2. $S \in \mathcal{C}(a) \Rightarrow \exists f \in \mathbb{R} f: C \rightarrow \mathbb{R}$

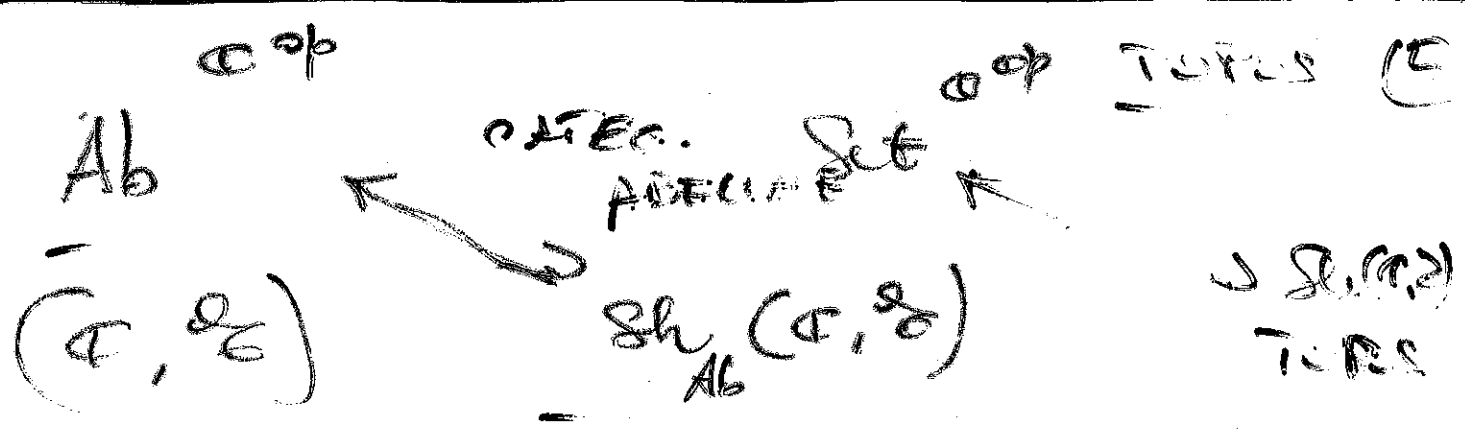


$\Rightarrow \bar{f} \in g^*(S) \neq \emptyset \Rightarrow g^*(S) \in \mathcal{C}(a)$

3. FINIRE PER EX

TOPOLOGIA ALCORICA

COSA SUCCEDERÀ PER $\tau = \mathcal{C}(X)$?



$P \in Sh_{Ab}(\mathbb{C}, \mathcal{O}_{\mathbb{C}}) \iff$

$P: \mathbb{C}^{\text{op}} \longrightarrow Ab \xrightarrow{U} \mathbb{R}^+$

P è un fascio $\mathcal{O}_{\mathbb{C}}$

AFFINE

TUTAS

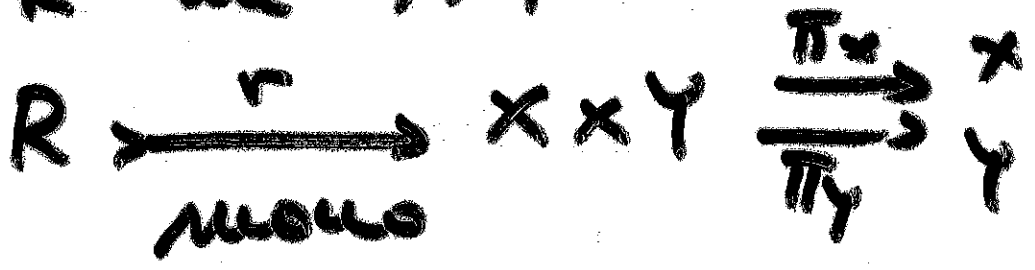
(RAPP.) ESATTA

Def Una categoria \mathcal{C} è (Bourbaki) esatta se

- LEX (LID FINITI)
- REGOLARE
- OGNI REL DI EQUIVALENZA È EFFETTIVA.

RELAZIONI in \mathcal{C} LEX

R tra $X, Y \in \text{ob } \mathcal{C}$

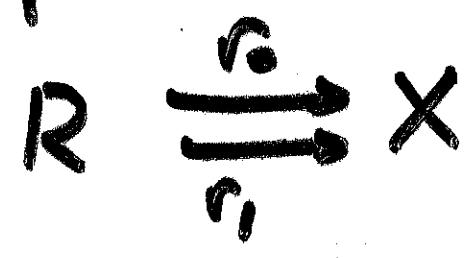


$$\pi_X \cdot r = r_0$$

$$\pi_Y \cdot r = r_1$$



$X = Y$ R rel su X



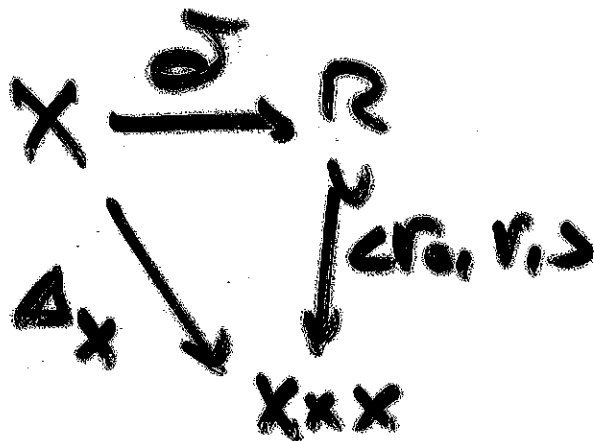
$\langle r_0, r_1 \rangle$ MONO

• RIFLESSIVA

(2)

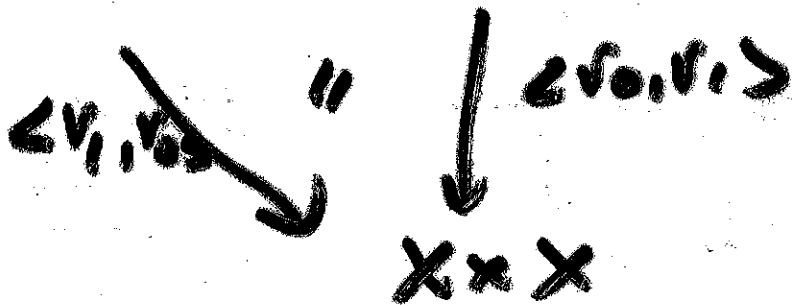
R è riflessiva $\Leftrightarrow \exists \sigma: X \rightarrow X$

$\text{cod } \sigma = \text{im } \sigma = X$



• SIMMETRICA $\Leftrightarrow \exists$

$\sigma: R \rightarrow R$

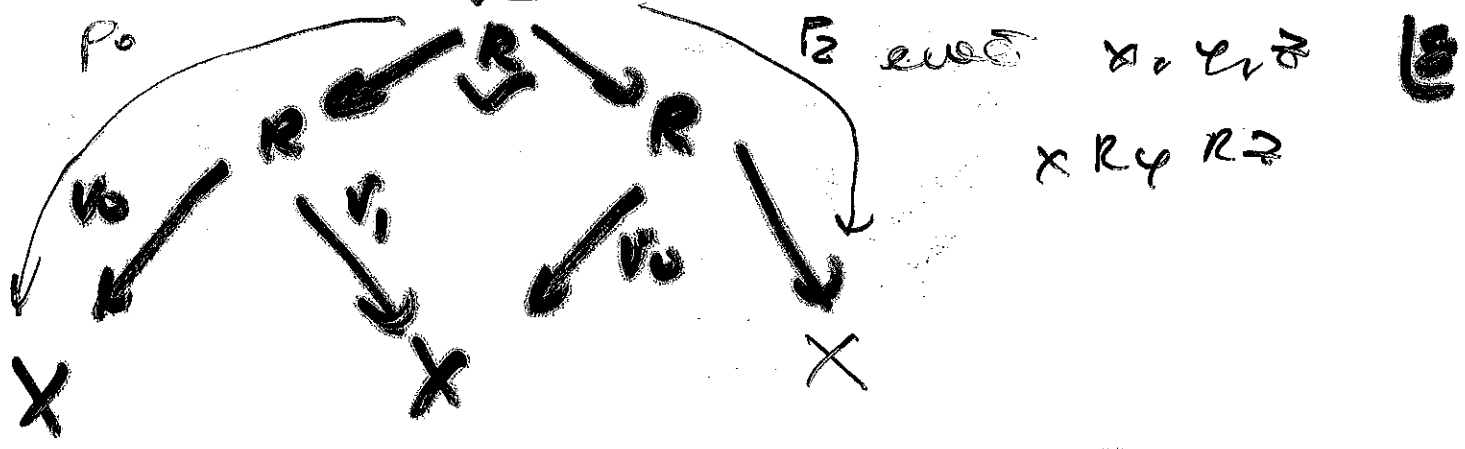


• TRANSITIVA

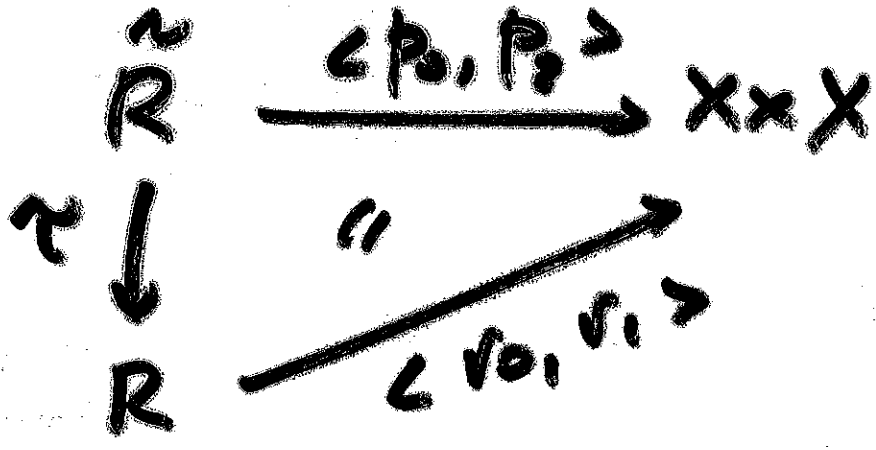
$xRy \wedge yRz \Rightarrow xRz$

~

xRy & yRz
~~implies~~



$\exists \tau$



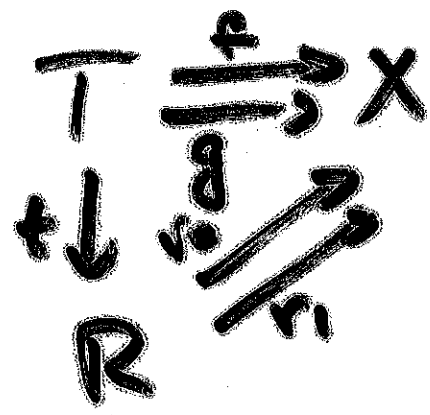
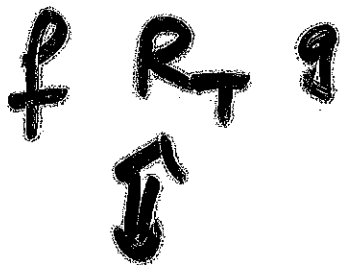
$p_0 \tau = p_0$
 $v_1 \tau = p_2$

Df R è di equis se
 rife, simm e trans.

$R \xrightarrow[v_1]{v_0} X \text{ val su } X$

$\forall T \in \text{ob } \mathcal{T}$, posso def
 una relazione R_T su

$\text{Hom}_{\mathcal{T}}(T, X)$ in Set



$\exists t$ l.c.
 $v_0 t = f$
 $v_1 t = g$

ex $R_T \bar{e}$ a) v.iff \forall
 b) s'null \top
 c) trans

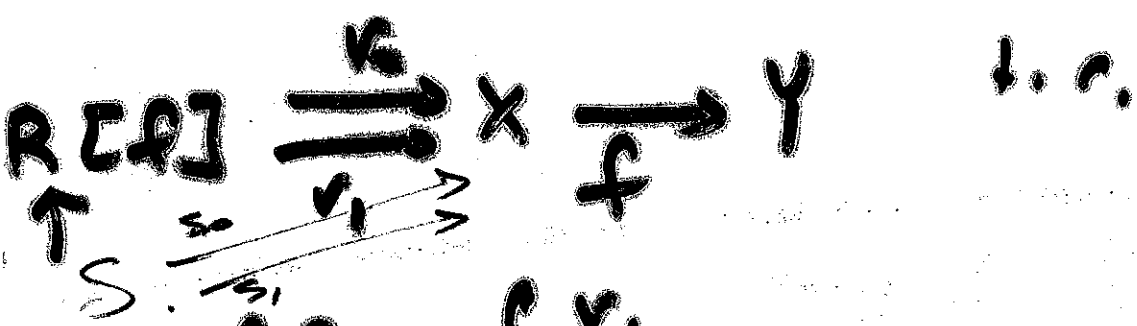
$\Leftrightarrow R \bar{e}$ a) b) c)

ESOPPI C ex

• $X \xrightarrow{\Delta_X} XXX$ DISCRETA

• $XXX \xrightarrow{\Delta} XXX$ INDISCRETA

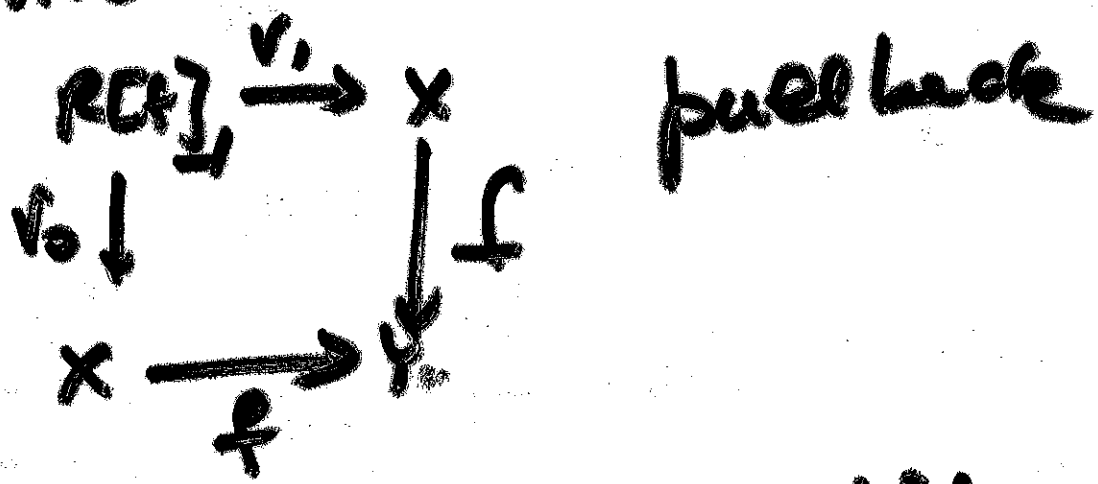
Kernel PAIR di una funzione



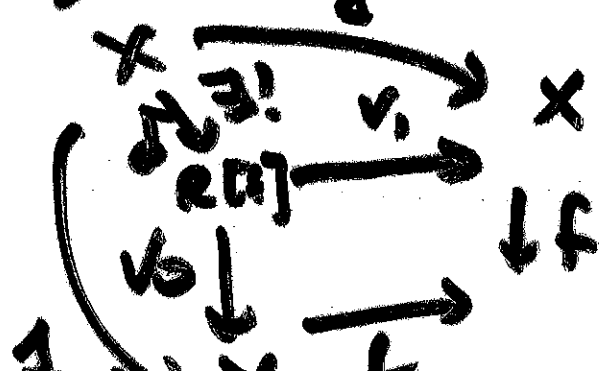
- $f \circ v_0 = f \circ v_1$
- $\exists f \circ v_0 = f \circ v_1 \Rightarrow \exists ! s$

$$S \longrightarrow \text{R[CP]} \quad \begin{array}{l} v_0 s = s_0 \\ v_1 s = s_1 \end{array}$$

si ottiene come



è una relazione di equivalenza:



è rifl!

FINIRE PER

EX

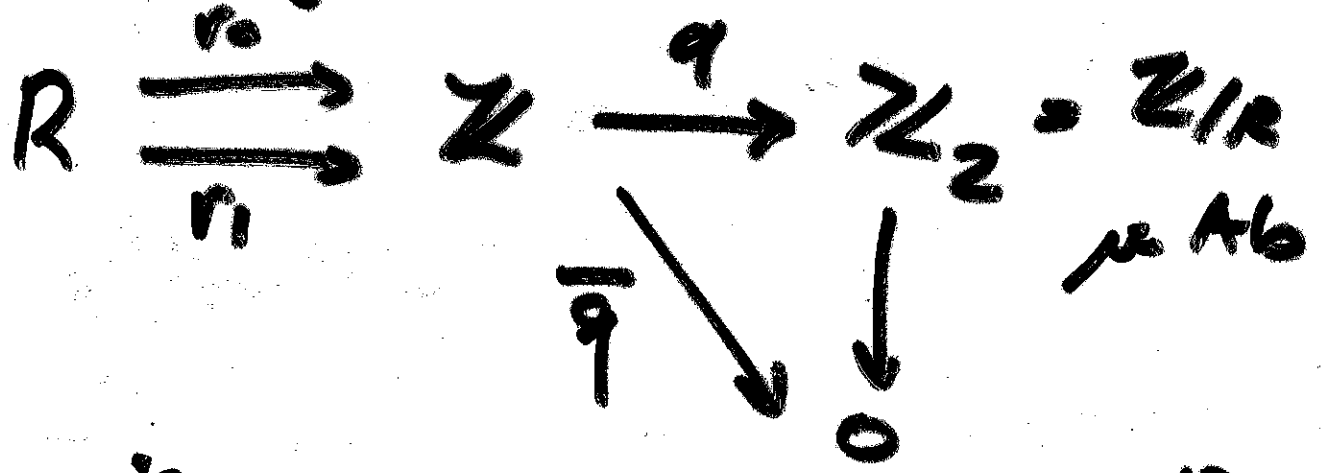
kernel pair = val d'equil
effettive

CI SONO RAE EQUIV NON
EFFETTIVE

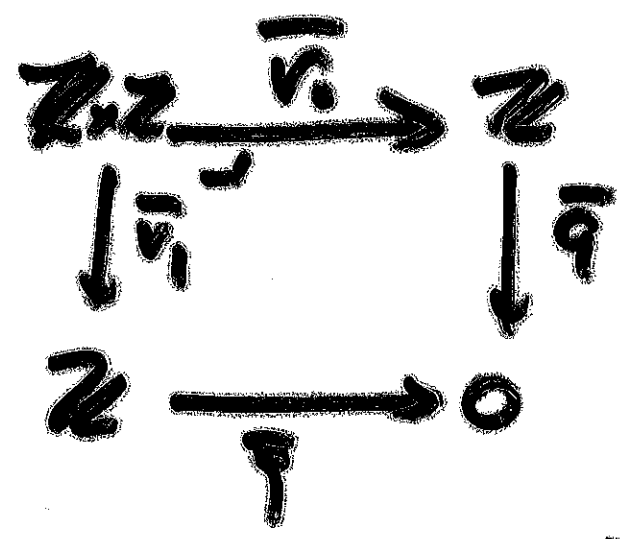
$\mathcal{C} = \text{Ab}_{\text{tf}}$ abelian libe
de torsione

$\mathbb{Z} \in \text{ob } \mathcal{C}$

$x R y \Leftrightarrow x - y = \text{pari}$



il quoziente \mathbb{Z} su \mathbb{R}
di \mathcal{C}



$$\bar{q} = \text{Coq}(M, N)$$

bonne pair et $\bar{q} \mathbb{Z} \times \mathbb{Z} \neq \mathbb{R}$

$$R \neq R[\bar{q}]$$

ex \bar{q} existe ie copresolutive

où $R \xrightarrow[\bar{v}_1]{\bar{v}_2} X$, alors R est effective $\Leftrightarrow R = R[\bar{q}]$

$\Rightarrow R \xrightarrow[\bar{v}_1]{\bar{v}_2} \mathbb{Z}$ non est

effective!

\Rightarrow Abs. est NON est BARR existe