

CT

6/10/2015

U

Df $F: \mathbb{C} \rightarrow \mathbb{D}$ è un'equivalenza
 & $FG: \mathbb{D} \rightarrow \mathbb{C}$, $\exists \eta: Id_{\mathbb{C}} \rightarrow GF$
 is naturale, $\exists \xi: Id_{\mathbb{D}} \rightarrow FG$

G è chiamata QUASI-INVERSA di F
 è anch'esso un'equivalenza ($\mathbb{D} \rightarrow \mathbb{C}$)
 • se F iso $\Rightarrow F$ equivalenza

Df \mathbb{C} e \mathbb{D} sono equivalenti
 (come categorie) se $\exists F: \mathbb{C} \rightarrow \mathbb{D}$
 equivalenza

$\mathbb{C} = \text{Set}_f$ insiemi finiti

$\mathbb{D} \hookrightarrow \mathbb{C}$ sottocategoria piena

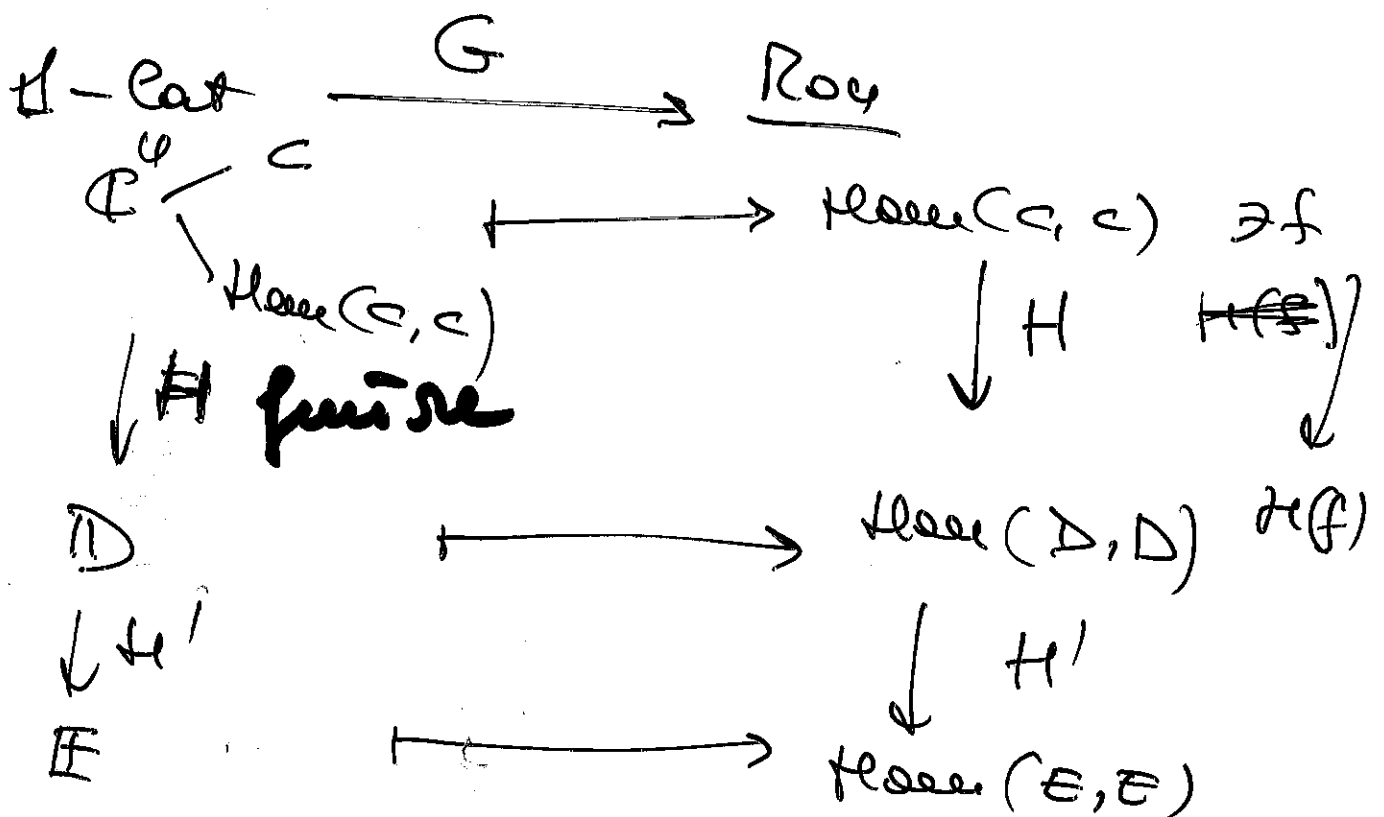
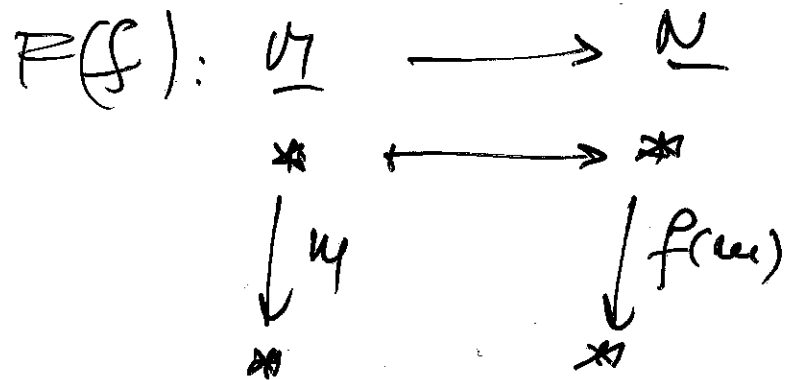
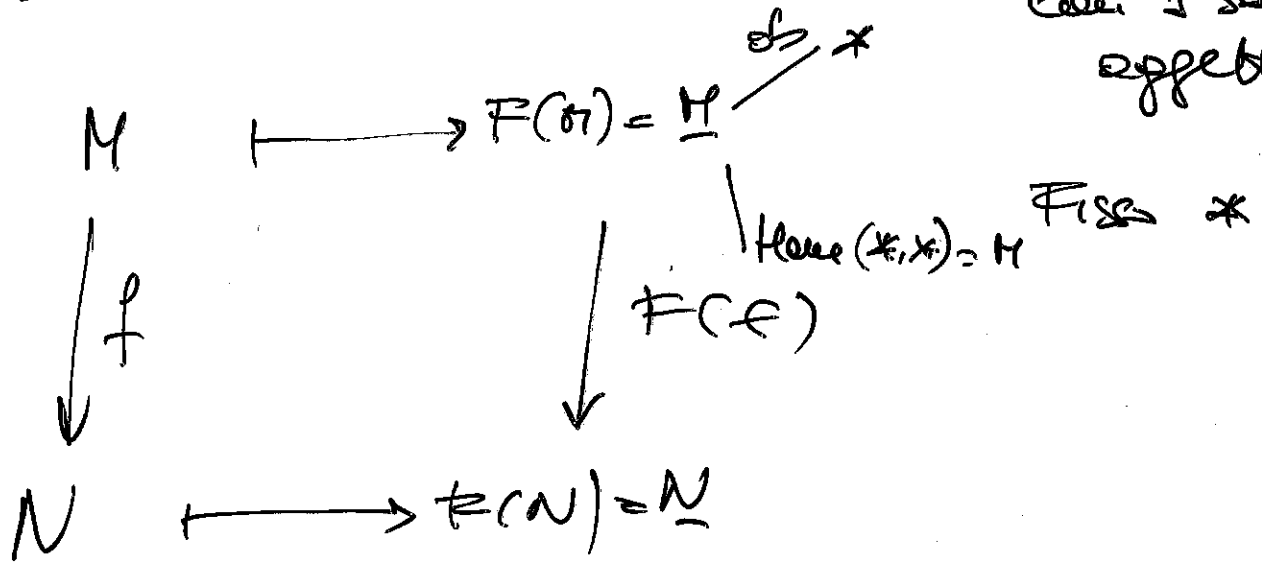
$$\text{ob}(\mathbb{D}) = \left\{ \begin{array}{l} \emptyset \\ \{0\} = \{0\} \end{array} \right.$$

$$\left\{ \begin{array}{l} \{0, 1\} = 2 \\ \{0, 1, \dots, n-1\} = n \end{array} \right.$$

\mathbb{D} è equivalente a \mathbb{C} (EX)

M mounable \longrightarrow \underline{M} cat & object

Hom \neq \Rightarrow $\underline{1}$ -cat = categorie
 con. 1 sol
 oggetti



$$\begin{array}{ccccc}
 \underline{\text{Hoep}} & \xrightarrow{F} & \underline{1\text{-Cat}} & \xrightarrow{G} & \underline{\text{Hoep}} \\
 M & \longrightarrow & \frac{M}{\downarrow} & \longrightarrow & \text{Hoep}(*, *) = \mathcal{A} \\
 & & \downarrow & & \\
 & & \mathcal{A}(x, M) & &
 \end{array}$$

$$G \circ F = \underline{1}_{\text{Hoep}}$$

$$\begin{array}{ccccc}
 \underline{1\text{-Cat}} & \xrightarrow{G} & \underline{\text{Hoep}} & \xrightarrow{F} & \underline{1\text{-Cat}} \\
 (C, \text{Hoep}(C, C)) & \longrightarrow & \text{Hoep}(C, C) & \longrightarrow & (*, \text{Hoep}(C, C))
 \end{array}$$

$$\text{FG} \xleftarrow{\eta} \underline{1}_{\text{1-cat}}$$

$$\begin{array}{l}
 \eta : C \longrightarrow \text{FG}(C) \\
 \eta : \text{1-cat} \longrightarrow \text{1-cat} \\
 \eta \text{ is natural!} \\
 \eta_C(C) = * \\
 \eta_C(f) = f \\
 \eta_C^{-1} : \text{FG}(C) \longrightarrow C \\
 * \longrightarrow C
 \end{array}$$

⇒ F epimorpha

Df $F: \mathcal{C} \rightarrow \mathcal{D}$ è essenzialmente
 suriettivo se e solo se

$$\forall D \in \mathcal{D}, \exists C_D \in \mathcal{C} \text{ e } \Sigma_D: C_D \rightarrow D$$

$$\Sigma_D \text{ iso}$$

$F: \mathcal{C} \rightarrow \mathcal{D}$ epimorfismo \Rightarrow

è essent. suriettivo se e solo se

$\forall D$, considero $GD \in \mathcal{C}$

e $\exists \Sigma_D: D \rightarrow FG D$ iso

con $C_D = GD$ $\Sigma_D = \Sigma_D^{-1}: F(GD) \rightarrow D$

$F: \mathcal{C} \rightarrow \mathcal{D}$ epimorfismo $\Rightarrow F$ è piena
 con un morfismo $G: \mathcal{D} \rightarrow \mathcal{C}$ e feabile

Dim \neq è feabile

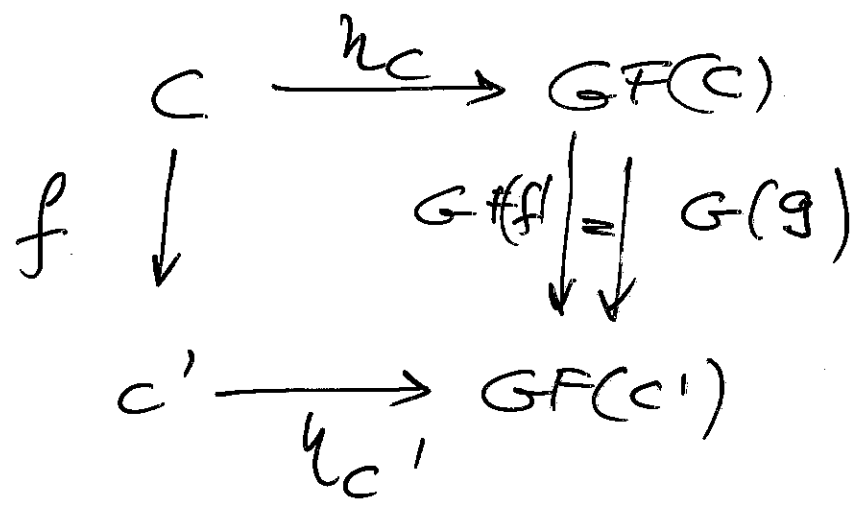
$$\begin{array}{ccc} C & & F(C) \\ f \downarrow & & \downarrow \\ C' & & F(C') \end{array} \quad \neq (f) \subset \neq (f') \Rightarrow f=f'$$

$$\begin{array}{ccc} C & \xrightarrow{h_C} & GF(C) \\ f \downarrow & \llcorner & \downarrow GF(f) \\ C' & \xrightarrow{h_{C'}} & GF(C') \end{array}$$

$$f = \eta_{c'}^{-1} \circ G(F(f)) \circ \eta_c$$

$$f' = \eta_{c'}^{-1} \circ G(F(f')) \circ \eta_c$$

F è pieno : data $F(c) \xrightarrow{g} F(c')$



$$\begin{aligned}
 f & := \eta_{c'}^{-1} \circ G(g) \circ \eta_c \\
 & \parallel \eta_c \\
 & = \eta_{c'}^{-1} \circ GF(f) \circ \eta_c
 \end{aligned}$$

G è fedele!
 $G(F(f)) = G(g)$
 \Downarrow
 $F(f) = g$

$\Rightarrow F$ è pieno (come per G)

$F \bar{e}$ agunt \Leftrightarrow libero
 fedele
 essenzialmente sur

Dire \Rightarrow OK

\Leftarrow Dobbiamo def: $G: \mathbb{D} \rightarrow \mathcal{C}$

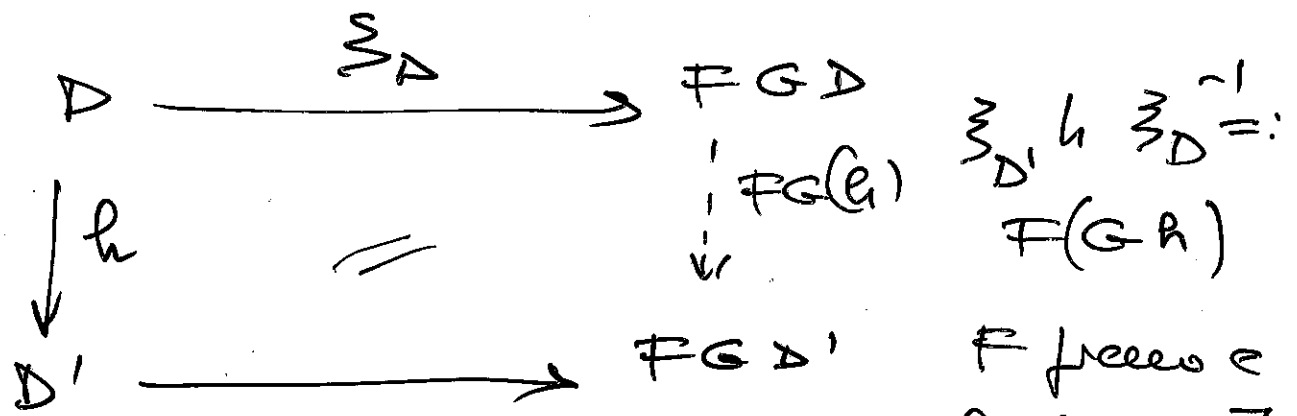
\downarrow : $\text{id}_{\mathcal{C}} \rightarrow GF$ \exists : $\text{id}_{\mathbb{D}} \rightarrow FG$

$\forall D \in \text{ob}(\mathbb{D}) \quad \exists C_D \quad \exists_D: C_D \xrightarrow{\text{iso}} D$

Df $G(D) = C_D$

$$\exists_D: D \xrightarrow{\text{iso}} FG(D) = FC_D$$

$\exists_D = \exists_D^{-1}$



$\exists_{D'}$
 G è def sulle free
 + naturalezze
 o \exists

F libero e
 fedele $\exists!$
 $G(h): GD \rightarrow GD'$
 h.c. $FG(h) =$
 $\exists_{D'} \circ h \circ \exists_D^{-1}$

EX G con di essere fedele! **17**

$$? \quad \eta : \text{Id}_G \longrightarrow GF$$

$$\forall \sum_{FC} : F(C) \xrightarrow{\text{iso}} F(GFC)$$

F piena e fedele $\Rightarrow \exists! \eta_c : C \rightarrow GFC$

$$\text{t.c. } F(\eta_c) = \sum_{FC} \text{iso} \Rightarrow \eta_c \text{ iso}$$

F riflette iso

EX η naturale □

F piena e fedele

$$\text{Hom}_G(C, C') \xrightarrow{(1.1)} \text{Hom}_D(FC, FC')$$

• $Ab \xrightarrow{\quad} Grp$ piena e fedele
non ess. surretto

Mat $\mathcal{A} = \{1, 2, \dots, n, \dots\}$

k campo 18

$$m \xrightarrow{A} n$$

$$A \in \text{Mat}(n \times m, k)$$

$$\downarrow B$$

$$p$$

~~Mat~~

~~Mat~~

$$B \circ A = B A$$

Mat \longrightarrow FD k -vet

$$n \longleftarrow k^n$$

$$\downarrow A \quad \downarrow A \cdot \vec{v}$$

$$m \longleftarrow k^m$$

ess. semelhante, preenche e fecha

\Rightarrow representação

An algebraic structure (L, \vee, \wedge) , consisting of a set L and two binary operations \vee , and \wedge , on L is a **lattice** if the following axiomatic identities hold for all elements a, b, c of L .

$$\begin{array}{lll}
 x \vee y = y \vee x & x \vee (y \vee z) = (x \vee y) \vee z & x \wedge (x \vee y) = x, \\
 x \wedge y = y \wedge x & x \wedge (y \wedge z) = (x \wedge y) \wedge z & x \vee (x \wedge y) = x
 \end{array}$$

A lattice (L, \vee, \wedge) is **distributive** if the following additional identity holds for all x, y , and z in L :

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z).$$

A **complemented lattice** is a bounded lattice (with least element 0 and greatest element 1), in which every element x has a **complement**, i.e. an element y such that

$$\begin{array}{l}
 x \vee y = 1 \quad \text{and} \quad x \wedge y = 0. \\
 y = \neg x
 \end{array}$$

A lattice (L, \vee, \wedge) is **complete** if *all* its subsets have both a join and a meet. In particular, every complete lattice is a bounded lattice

A Boolean algebra or Boolean lattice is a complemented distributive lattice.

An **atom** of a Boolean algebra is an element x such that there exist exactly two elements y satisfying $y \leq x$, namely x and 0 .

A Boolean algebra is said to be **atomic** when every element is a sup of some set of atoms

(10)

A **Boolean ring** R is a ring for which $x^2 = x$ for all x in R

Given a Boolean ring R , for x and y in R we can define

$$\begin{aligned}x \wedge y &= xy, \\x \vee y &= x + y + xy, \\ \neg x &= 1 + x.\end{aligned}$$

These operations then satisfy all of the axioms for meets, joins, and complements in a Boolean algebra. Thus every Boolean ring becomes a Boolean algebra.

Similarly, every Boolean algebra becomes a Boolean ring thus:

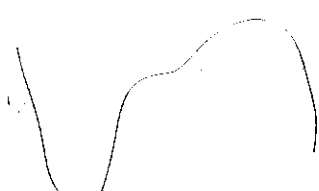
$$\begin{aligned}xy &= x \wedge y, \\x + y &= (x \vee y) \wedge \neg(x \wedge y)\end{aligned}$$

without changing the underlying set

A map between two Boolean rings is a ring homomorphism if and only if it is a homomorphism of the corresponding Boolean algebras, that is a function $f: A \rightarrow B$ such that for all x, y in A :

$$\begin{aligned}f(x \vee y) &= f(x) \vee f(y), \\f(x \wedge y) &= f(x) \wedge f(y), \\f(0) &= 0, \\f(1) &= 1.\end{aligned}$$

This way we construct an isomorphism between the category of Boolean algebras and the category of Boolean rings



DUALITA' di STONE

Boole Alg equiv. (Stone Sp)^{op}

compati Haus
totalmente
discreti

Coupl Atomic Boolean Algebra

u

CABA equivaleto $(Set)^{op}$ CABA equivalente

