

~~TC~~

~~10/11/2015~~

Def  $T$  monade decompuncta se

$\mu_x$  e iso  $\forall x$

$(\Rightarrow (X, \xi) \in \mathcal{C}^T \cong \text{iso})$



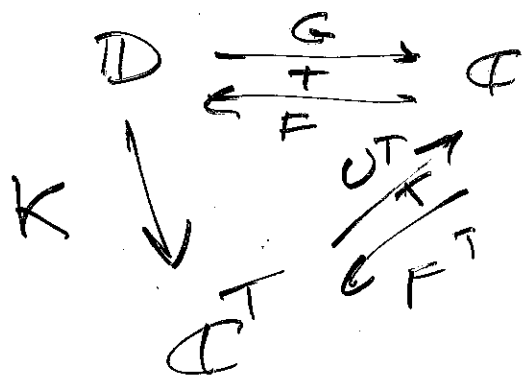
la comuta di queste approssime la  
 come componenti  $(X, \xi) \xrightarrow{\quad} = \xi : TX = F^T U^T X \rightarrow X$   
iso

$\Downarrow$   
 $U^T$  e pieno e fedele

e  $F \dashv G : \mathcal{D} \rightarrow \mathcal{C}$   $G$  pieno e fedele

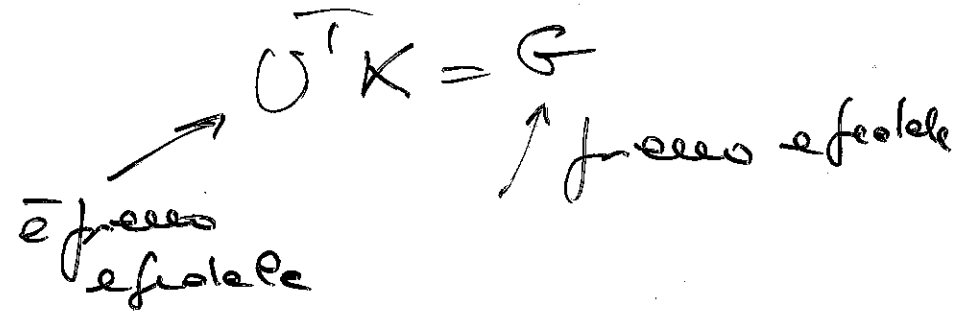
$T = GF$  e una monade decompuncta  
 fante?  $\mu_x = G(\epsilon_{FX})$   
 $\uparrow$  iso  
iso

$\Rightarrow T$  e decompuncta  
 molte



$$T = GF \quad \text{Ⓢ}$$

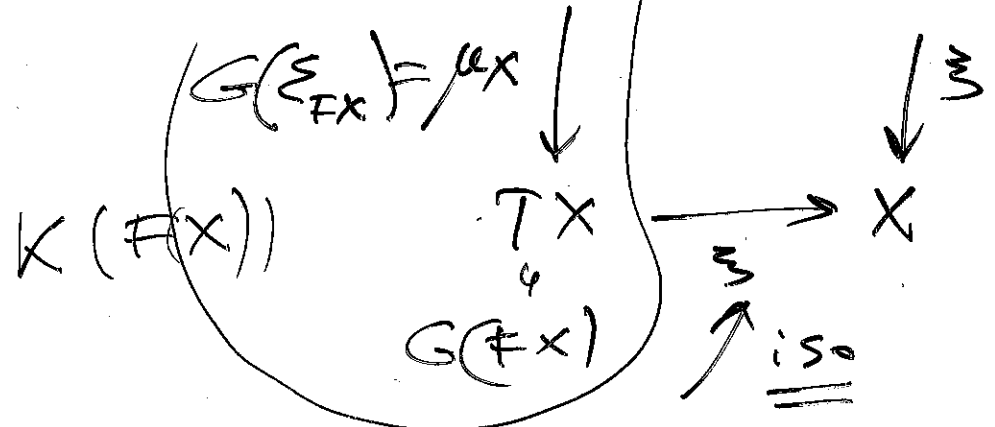
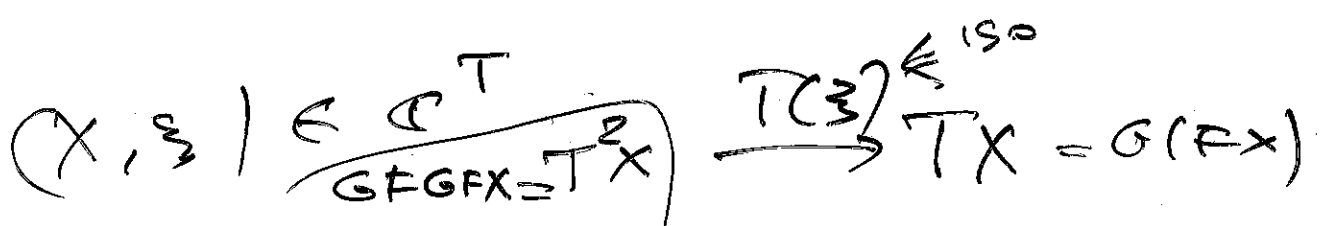
$\exists K$  cofactors  
coe



$\xRightarrow{EX} K$  free e fedele

ma  $K$  è anche essenzialmente  
 suriettivo:  $\forall (X, \mathbb{E}) \in \mathbb{C}^T$   
 $\exists Y \downarrow c. K(Y) \cong (X, \mathbb{E})$

$$K(Y) = (G(Y), G(\mathbb{E}_Y))$$



$T$  idemp.

$$K(F(X)) = (FX, G(\mathbb{E}_{FX})) \cong (X, \mathbb{E})$$

$\Rightarrow G$  è monomorfico

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$T$  monomorfico ideale  $\xleftrightarrow{(1,1)} F + G: D \rightarrow C$   
 $G$  fedele e fedele (monomorfico)

$\swarrow \searrow$   
 $a-d$  epimorfismo

SOTTOCATEGORIA RIFLESSIVA (di  $C$ )

$J: A \hookrightarrow C$  SOTTOCATEGORIA PIENA  
 di  $C$  si dice RIFLESSIVA

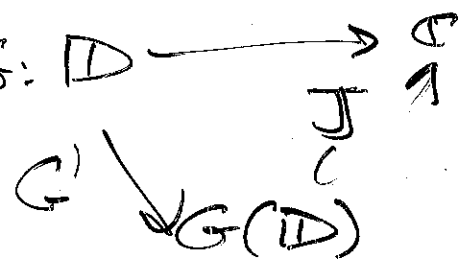
$J$  ha aggiunto sinistra  $R: C \rightarrow A$

$R + J$  ( $J$  è fedele e fedele)

Se fatto da  $F + G: D \rightarrow C$   
 con  $G$  fedele e fedele, trono che

$\mathbb{D}$  è epimorfismo a una sottocategoria  
 (riflessiva) di  $C$   $G: D \rightarrow C$

$G'$  è fedele, fedele  
 (ess.) suriettivo  $\Rightarrow G'$  epimorfismo





$$\text{Abel} \xrightarrow{UJ} \underline{\text{Set}}$$

$$\underline{\text{Set}} = \text{Set}^{\mathbb{T}} \cong \underline{\text{Ab}}$$

$K$  mon è esse' epimorfismo

$\Rightarrow UJ$  mon è monomorfo

MONADI

See  $\mathbb{C}$

$$(T, \mu, \eta)$$

$$T \in \text{ob}(\text{End}(\mathbb{C}))$$

oppelli:  
 leleofunctor  
 murfz

$$\mu: T^2 = T \circ T \xrightarrow{\text{e.u.}} T$$

$$\eta: \text{Id}_{\mathbb{C}} \xrightarrow{\text{I}} T$$

ASSOCIATI

ASSOCIATIVITA' +  
 elemento neutro  
 a d s e a s m

Monoidi  
 e s m

$$(M, m, e)$$

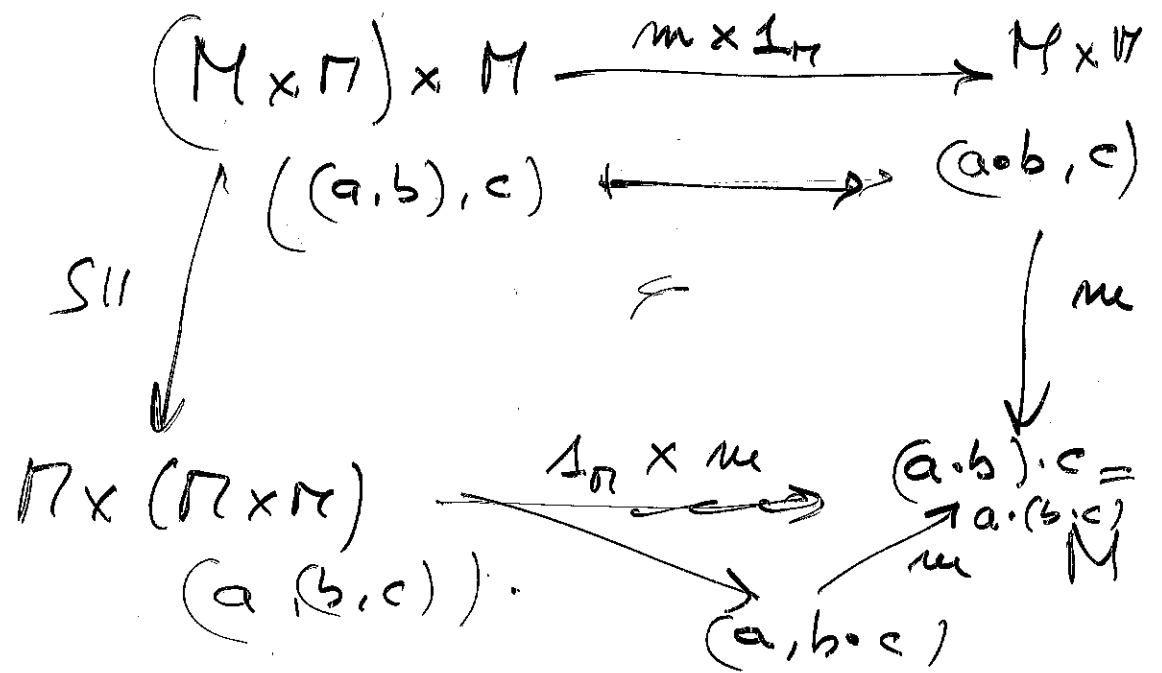
$$M \in \text{ob}(\text{Set})$$

$$m: M \times M \longrightarrow M$$

$$e: \{*\} \longrightarrow M$$

I

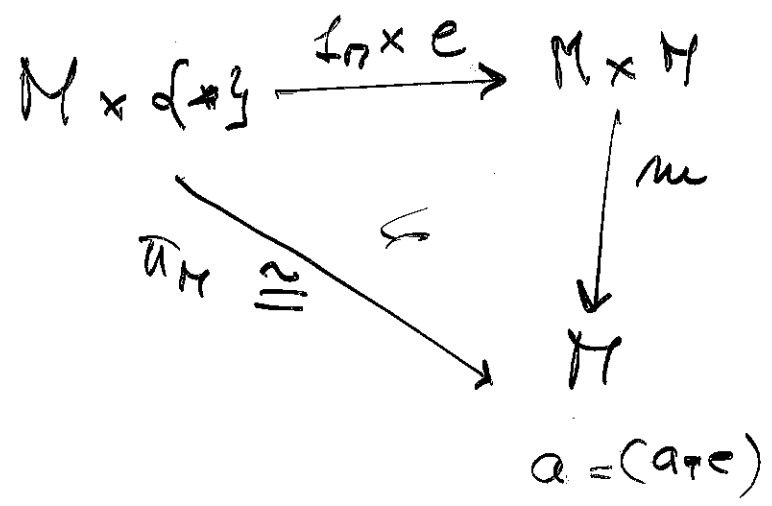
$e(*) = e$



elementary matrices  $\mu$

$$M \times M \xrightarrow{\mu} M$$

$(a, e)$   $a \circ e$



+ also ax

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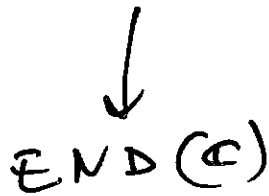


# CATEGORIE MONOIDALI

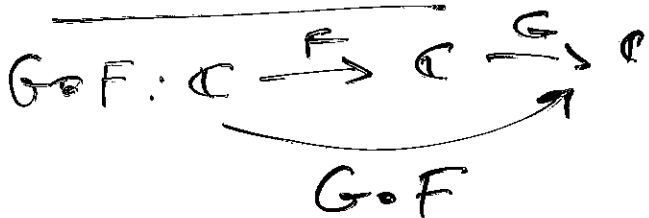
$(\text{END}(\mathcal{C}), \circ, \frac{1}{\mathcal{C}} = I)$   
 monoidale stretta

$(\text{Set}, \times, \text{funct}) = I$   
 + "no commutative"  
 monoidale

$- \circ - : \text{END}(\mathcal{C}) \times \text{END}(\mathcal{C})$

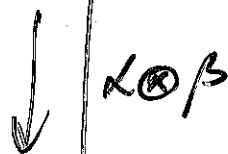
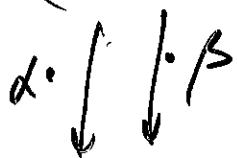


BI FUNTORE



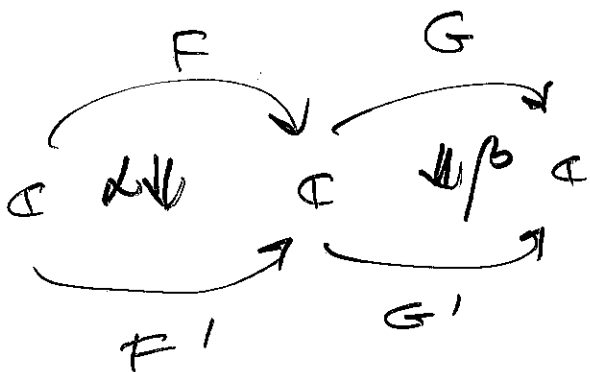
Def  $\otimes F \otimes G = G \circ F$

$$(F, G) \longmapsto F \otimes G = GF$$



$$(F', G') \longmapsto F' \otimes G' = G'F'$$

$- \otimes -$  è  
 un bifunctor



$\alpha \otimes \beta = \text{comp. orizzontale}$   
 di  $\alpha$  e  $\beta$

$$\alpha \otimes \beta : GF \longrightarrow G'F'$$

$(\alpha \otimes \beta)_C$

$$GF \xrightarrow{G(\alpha_C)} G'F' \xrightarrow{G'(\beta_C)} G'F'G'$$

$$I := \text{Id}_F \quad F \otimes I = I \otimes F = F$$

$$(F \otimes G) \otimes H = F \otimes (G \otimes H)$$

$$(x \otimes y) \otimes z = x \otimes (y \otimes z)$$

$(\text{End}(F), \otimes, I = \text{Id}_F)$  è una  
algebra di cat. necessaria da studiare

Df  $(V, \otimes, I)$  è una cat.  
monoidale stretta con

$$-\otimes- : V \times V \longrightarrow V \quad \text{bifunctor}$$

$$I \in \text{ob}(V)$$

Ad: che

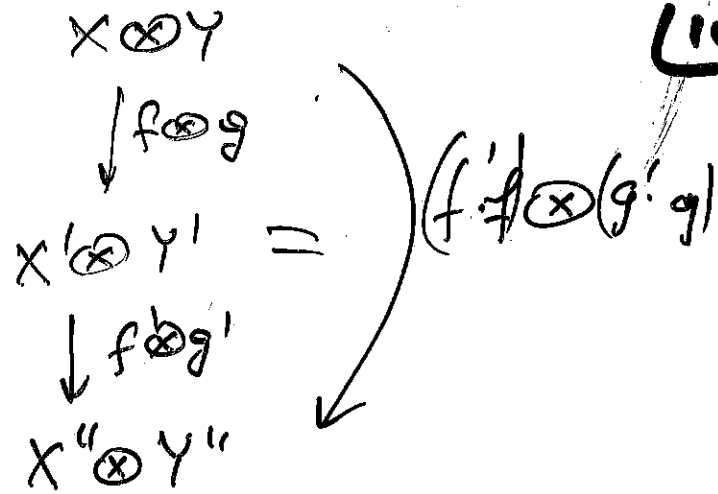
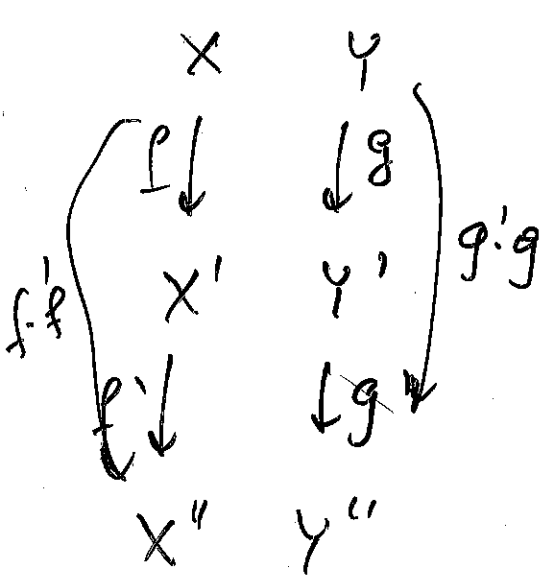
$$(X \otimes Y) \otimes Z = X \otimes (Y \otimes Z)$$

$$(f \otimes g) \otimes h = f \otimes (g \otimes h)$$

$$I \otimes X = X \otimes I = X$$

$$1_I \otimes f = f \otimes 1_I = f$$

perché  $-\otimes-$  è un bifunctor



$$(f' \otimes g') \circ (f \otimes g) = f' \otimes g'$$

$$1_X \otimes 1_Y = 1_{X \otimes Y}$$

ES

- $(\text{End } \mathbb{C}, \otimes, I = \text{Id}_{\mathbb{C}})$
- $\mathbb{M}$  see monoids visto come categoria DISCRETA

$$m \otimes m' = m m' \quad I = e$$

- $\mathbb{D} = \text{oggetti di } \mathbb{D}$
- $\forall n \geq 1 \quad \underline{n} = \{1, \dots, n\}$  numeri
- $n = 0 \quad 0 = \emptyset \quad f : \underline{n} \rightarrow \underline{m}$
- $f$  monotone

$$n \otimes m = n + m = \{1 \dots n, n+1 \dots n+m\}$$

$$I = 0$$

$$f : \underline{u} \longrightarrow \underline{u}$$

$$g : \underline{n'} \longrightarrow \underline{u'}$$

$$f \otimes g : n \otimes n' \longrightarrow u \otimes u'$$

(Set,  $\times$ ,  $\Delta^*? = I$ ) non è stretta

$$A \otimes B = A \times B = \{(a, b) \mid a \in A, b \in B\}$$

$$(A \otimes B) \otimes C \neq A \otimes (B \otimes C)$$

non sono isomorfi!

$$A \otimes I = A \times \{*\} \neq A$$

è però una categoria  
 monoidale (non stretta)

Def Una categoria monoidale  
 è data da una cart  $\mathcal{V}$

con

$$\otimes : \mathcal{V} \times \mathcal{V} \longrightarrow \mathcal{V} \text{ funzione}$$

$$I \in \text{ob}(\mathcal{V})$$

$$A$$

$$x, y, z$$

$$\alpha_{x, y, z} : (x \otimes y) \otimes z \xrightarrow{\sim} x \otimes (y \otimes z)$$

naturale in  $x, y, z$

$\alpha$   
 $X$

$$\lambda_X : I \otimes X \xrightarrow{\cong} X$$

naturale in  $X$

$\rho$   
 $X$

$$\rho_X : X \otimes I \xrightarrow{\cong} X$$

naturale in  $X$

$$(U, \otimes, \alpha, \lambda, \rho)$$

