DESCRIPTION OF CURRENT RESEARCH INTERESTS

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I am pursuing a rigorous Renormalization Group approach based on multiscale analysis and convergent expansions, see e.g. [1] for a general introduction. Convergence is essential to exclude the presence of non-perturbative phenomena and to rigorously establish universality properties, and in the case of fermions is based on sign cancellations between Feynman graphs in the form of determinant bounds, cluster and tree expansion and emerging or exact Ward identities. The generality of this approach makes possible to apply it to a wide range of physical phenomena, exploring and taking advantage of strict connections in the mathematical structure between apparently unrelated fields, allowing to use ideas from a field to another.

1. Universality in Hall conductance and Graphene transport coefficients

Several transport coefficients in Condensed Matter show remarkable universality properties, appearing in an insensitivity of such coefficients to the details of the interaction. Well known examples include the quantization of the Hall conductance, which is so precise to be used to determine the fine structure constant, and more recent ones include the optical conductivity in Graphene [2].

Universality in graphene found experimentally was in contradiction with previous perturbative computations predicting a strong renormalization of the conductivity, see e.g. §7.3 of [3] for a short review of the problem. Together with my collaborators we proved a theorem [4],[5], based on a rigorous Renormalization Group analysis [6] finally establishing the absence of interaction corrections to the conductivity in a system of fermions on the honeycomb lattice with a weak short ranged interaction. Universality seen in experiments came from delicate cancellations between the irrelevant terms produced by the lattice, following from regularity properties of the correlations. Universality is present in the conductivity and not in other quantities like the velocities. At the moment, this is the only rigorous proof of universality in graphene, and it requires non-perturbative bounds on the decay of correlations obtained by multiscale analysis. The proof reveals clear analogies with the property of the non-renormalization of the anomalies in Quantum Field Theory. An interesting open problem, on which I am currently working, is to prove (if possible) a similar theorem in the case of Coulomb interaction, or to get non perturbative information on the behavior of the conductivity as function of the temperature or frequency.

The proof of quantization of the Hall conductance for interacting electrons in the thermodynamic limit was a well known problem, present in the list of fundamental open problems in mathematical physics in [7]. Indeed topological arguments explain the quantization of the conductance in the non interacting case, but a proof that quantization survives in presence of interaction was lacking. This well known conjecture was finally proved by myself and collaborators in [9] using multiscale analysis. While the result was also found simultaneously by other groups [8] the method of our proof is much more general. It was indeed used to provide the proof of another well known conjecture in the interacting case, the so called bulk-edge correspondence ; the proof can be found in [10],[11] and is actually the only rigorous proof of such a property. Open problems on which I am currently working is the interplay of disorder and interaction, and in particular if universality persists; and in perspective to face the fractional Quantum Hall effect from a microscopic point of view.

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2. Constructive Quantum Field Theory and Anomalies

Very few QFT, in $d = 1 + 1$, have been mathematically constructed at a non-perturbative rigorous level using functional integrals, see e.g. [12] for a recent review, and typically they are superrenormalizable or asymptotically free [13]. At the moment the only example with non gaussian fixed point is provided by the construction, done by myself and collaborators, of the massive Thirring model in [14], [15] describing Dirac fermions with a current-current interaction. It was proved that a lattice functional integrals for Euclidean correlations is well defined taking both the continuum and the infinite volume limit; in addition Osterwalder–Schrader axioms are verified so that a QFT in the sense of Wightman is constructed. The presence of a line of fixed points require the implementation of Ward Identites at each step of the RG, allowing to control the flow of the running coupling constants. This was possible developing a method allowing to resolve the well known contradiction between WI based on local symmetries and the use of Wilsonian RG methods. The momentum cut-off breaks the WI producing corrections, and a new technique was developed in order to control such corrections present at each step. In addition, this approach allows to establish rigorously also the presence of chiral anomalies and to prove the non-perturbative validity of the Adler Bardeen theorem in d=1+1. By extending such methods, a proof of the Coleman equivalence with the Sine-Gordon model was given [16]. Such technique found a number of applications in universality properties of condensed matter or statistical physics [17].

More recently, I have started to apply such methods to $d = 3 + 1$ with a finite lattice, with the long term goal of constructing an anomaly free lattice version of the Electroweak sector of the Standard Model with a cut-off exponentially high in the inverse coupling. A basic prerequisite is that in a lattice regularization of such theory the chiral anomaly cancel out with a finite lattice under the anomaly cancellation condition. This requires the proof of an anomaly non-renormalization theorem in a non-perturbative context and at finite lattice. In [18], [19] a non-perturbative proof of the Adler-Bardeen theorem has been achieved in a (massive) QED lattice model, using lattice Ward Identities and regularity properties of the correlations following by convergent expansions. In [20], [21] the anomaly cancellation has been proven in chiral gauge theories with cut-off of the order of the inverse coupling; remarkably the same condition found at one loop ensure the non-perturbative validity of the cancellation. Non-perturbative effects are excluded.

3. Universality and critical phenomena

The critical properties, and in particular the exponents in statistical mechanics, show remarkable universality properties in the sense of independence from microscopic details. In particular adding a perturbation to the 2D Ising model, like a next to nearest neighbor interaction, the exponents are expected to be the same as in the Onsager solution. This is not true adding a perturbation to coupled Ising models, like vertex or Ashkin-Teller models, in which the exponents depend on details; however it is expected that the exponents verify exact Kadanoff relations [22] allowing the determination of the exponents in terms of a single one of them. Such relations can be proved in the case of rare solvable models but a general mathematical proof was lacking despite intense research.

In [23] a new approach was proposed starting from the Grassmann integral representation at the basis of the Onsager solution, which leads to the expression of the energy exponents in terms of convergent series in the case of coupled Ising and to the proof of universality for a perturbed Ising model. In contrast with the usual high and low temperature series expansions used in statistical physics, the series are convergent up to the critical point. In [24], [25], [26], the Kadanoff relations were finally proved for the energy and correlation length exponents. Subsequenlty the technique has been applied to the proof of the universality of the central charge in perturbed Ising model [27], [28]. In [29], [30] it was considerd the related interacting dimer model and it was proved the Haldane universality conjecture for the height function.

Multiscale methods have been then applied in dimension higher than 2 [31] with the aim of providing a rigorous foundation to the epsilon expansion and proving the existence of a non gaussian fixed point, and in perspective to prove the analyticity of exponents.

Similar techniques have been adopted to prove universality properties of the zero temperature Drude weight in non integrable quantum spin chain like perturbations of the XXZ spin chain; by combining emerging and lattice Ward identities and convergence of the expansion in [32] was proved that the Drude weight is non vanishing and universal either for integrable and non integrable systems. It has been however conjectured that at finite temperature the Drude weight has a different behavior for integrable and non integrable perturbations; in particular it should be non vanishing for integrable and vanishing for non integrable. This conjecture requires a deep analysis of the cancellation from the irrelevant terms in the RG sense and I plan to apply the above methods to the problem.

4. Small divisors and Many Body Localization

The issue of the persistence of localization in presence of disorder is widely studied; numerical results present ambiguities and a mathematical proof in [34] in the case of random disorder relies on unproven assumptions; there are no firm results even in the case of zero temperature. Actually most experiments are done with quasi-periodic potentials [33] with cold atoms.

I gave the only existing complete mathematical proof of localization in an interacting fermionic system at T=0 with interaction and strong quasi-periodic potential, see [38], [39], [40]. The idea of the proof is to combine the direct methods developed in classical mechanics and KAM (Kolmogorov Arnold Moser) theory [35],[36],[37], for dealing with small divisors with the RG techiques used to deal with interacting fermions, In particular one needs number theoretical conditions on the frequency (diophantine conditions) and cancellations due to Pauli principle. Note that, as in the case of Birkoff series in classical mechanics, the existence or not of localization cannot be understood by perturbative computations but it relies on the convergence or divergence of the series. An open issue which I plan to study is to extend this approach to finite temperature.

In the case of weak quasi periodic potential instead there is no localization, as proven in [41] in one dimension, for smooth potential, see also [42]. In the case of the 3d dimensional Dirac semimetals with quasi-periodic disorder, stability was numerically observed in [43] and rigorously proved in [44], again controlling the small divisors via direct KAM techniques; similar methods are expected to be applied to the still open and debated case of random disorder in Weyl semimetals.

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