Integrali impropri #2

(Marco Vignati 16/17)

1] Stabilire se i seguenti integrali generalizzati sono convergenti:

a) $\displaystyle\int_0^{+\infty} \frac{\log x}{(x-1)\sqrt{x}}\,dx$

b) $\displaystyle\int_0^{+\infty} \frac{\log(x+1)}{(x-1)\sqrt{x}}\,dx$

c) $\displaystyle\int_1^{+\infty} \frac{x}{(x^2+5)^{3/2}}\,dx$

d) $\displaystyle\int_3^{+\infty} \frac{dx}{3x^2+5x+2}$

e) $\displaystyle\int_0^{1} \frac{e^x-1}{x\sqrt{x}}\,dx$

f) $\displaystyle\int_2^{+\infty} \frac{x\sqrt{x+1}}{x^2-1}\,dx$

g) $\displaystyle\int_0^{+\infty} \frac{(x^2-\arctan^2 x)\,\sqrt[5]{(x^2-1)^6}}{(x^6+2x^5)\,\log^2 x}\,dx$

2] Stabilire per quali valori $a \in \mathbb{R}$ i seguenti integrali generalizzati convergono:

a) $\displaystyle\int_0^{+\infty} \frac{\log(1+t^a)}{\arctan t\,+\,2t^{2a}}\,dt$

b) $\displaystyle\int_0^{+\infty} \frac{\log(1+t^2)}{|t-a|^a\,\sqrt{t^5}}\,dt$

c) $\displaystyle\int_0^{1} \frac{\sin(\pi t)}{t^2-a}\,dt$

d) $\displaystyle\int_a^{+\infty} \frac{e^{at}\,|3t+1|^a}{2+e^{(3-a)t}}\,dt$

e) $\displaystyle\int_{1-a}^{a^2-1} \frac{e^{-t}}{t+1-a}\,dt$

Risposte:

1] Due sono divergenti, cinque sono convergenti. Quali?

2] In ordine sparso:

$a \in \left(-\dfrac{3}{2}, 1\right)$; $\quad a \in \left(-1, \dfrac{3}{2}\right)$; $\quad a \in (-\infty, 0) \cup \{1\}$;

$a \in (-\infty, -1) \cup \left(\dfrac{1}{2}, +\infty\right)$; $\quad a \in (-\infty, 0) \cup [1, +\infty)$