From semi-classical to quantum many body through normal forms

Milano, December 17-20, 2019

Titles & Abstracts

Mini courses:

Benjamin Schlein: *Bogoliubov theory for Bose-Einstein condensates*

We consider systems of N bosons in the Gross-Pitaevskii regime. We present a rigorous version of Bogoliubov theory and we explain how it can be used to show the emergence of Bose-Einstein condensation and to compute the ground state energy and the excitation spectrum, up to errors vanishing as N tends to infinity. We will conclude with some recent results that are valid beyond the Gross-Pitaevskii limit.

San Vũ Ngọc: *Quantum integrable systems and asymptotic lattices*

In the first part I will present an overview of the inverse spectral problem for quantum integrable systems. I will explain the case of 1D Hamiltonians, and the relationship between the quantum spectrum and the topological Reeb graph of the energy level sets. Then I will present more general questions arising in higher dimensional quantum integrable systems, and why we have been recently led to introduce the notion of asymptotic lattices (joint work with Monique Dauge and Mike Hall).
In the second part I will give more details and proofs. In particular, I will explain how to prove Bohr-Sommerfeld rules using microlocal quantum normal forms, and how asymptotic lattices allow to recover the integral affine structure from the spectrum. Very recently, a new application to semitoric systems was found (joint work with Yohann Le Floch).

Talks:

Gianfausto Dell'Antonio: *Contact interactions and Gamma convergence*

Fabricio Macià: *Concentration and non-concentration of modes and quasi-modes*

In this talk we present results on the structure of high-energy eigenstates of Schrödinger operators. We will mostly focus on operators that are perturbations of completely integrable systems (such as the Laplacian on the torus or a Zoll manifold, or the Quantum Harmonic Oscillator). Our main goal is to understand the effect of the perturbation on the high-frequency behaviour of eigenstates, and more precisely, in characterising the set of semiclassical defect measures associated to sequences of eigenfunctions.
I will present joint work with Victor Arnaiz (ICMAT, Madrid) and Gabriel Riviere (Nantes) on these problems.

Antonio Ponno: *Coherent dynamics in large size Bose-Hubbard models*

Gueorgui Popov: *KAM theorems quasimodes and applications*

Given a smooth one parameter family of Hamiltonians close to a Kolmogorov nondegenerate completely integrable Hamiltonian, we prove the existence of smooth one parameter families of KAM tori and provide global estimates in the scale of Hölder spaces. This leads to a construction of smooth one parameter families of quasimodes for the corresponding semiclassical operators. Applications to spectral rigidity of Laplace-Beltrami operators are obtained. These quasimodes can be used as well to extend a recent result of Gomes and Hassell about the existence of semiclassical measures associated to the eigenfunctions of semiclassical operators that have positive mass on KAM tori.

Marcello Porta: *On the correlation energy of mean field Fermi gases*

In this talk I will discuss the ground state properties of a homogeneous, interacting Fermi gas, in the mean-field regime. I will focus on the correlation energy, defined as the difference between many-body and Hartree-Fock ground state energies. It is a long-standing open problem in mathematical physics to rigorously compute this quantity, for large quantum systems. I will present upper and lower bounds for the correlation energy, that are optimal in their dependence on the number of particles, and

that agree for small interactions. The lower bound captures the corrections to the energy predicted by second-order perturbation theory; it is based on the combination of Bogoliubov theory and on correlation inequalities for the many-body interaction. The upper bound establishes the validity of the random-phase approximation as a rigorous upper bound to the ground state energy; it is based on a suitable choice of the trial state, and on a rigorous bosonization scheme.
Talk based on joint works with C. Hainzl and F. Rexze, and with N. Benedikter, P. T. Nam, B. Schlein and R. Seiringer.

Michela Procesi: *Reducibility for a Schrodinger operator with almost-periodic potential*

We study the reducibility of a Linear Schrodinger equation subject to a small unbounded almost-periodic perturbation which is analytic in time and space. Under appropriate assumptions on the smallness, analiticity and on the frequency of the almost-periodic perturbation, we prove that such an equation is reducible to constant coefficients via an analytic almost-periodic change of variables. This implies control of both Sobolev and Analytic norms for the solution of the corresponding Schrodinger equation for all times.

Didier Robert: *Quantum Propagation of Wave Packets for codimension one crossings eigenvalues*

We consider a time dependent Schrödinger equation for systems in the semi-classical regime. The Weyl symbol of the Hamiltonian has a matrix structure and its eigenvalues determine the modes of the system. We consider the case of a data localized along an isolated mode and the situation where this mode intersects smoothly with another one. This relies on the precise study of the propagation of coherent states through codimension one eigenvalue crossings and on Gaussian initial value representation of the initial data. Our results extend and clarify previous works of G. Hagedorn and of A. Watson and M. Weinstein. Our aim is to obtain an Herman-Kluk formula for the propagator of a semi-classical system of Schrödinger equations. This a joint work, in progress, in collaboration with Clotilde Fermanian-Kammerer and Caroline Lasser.