Numerical methods for PDEs 1

13.11.2018 – problem set n. 6

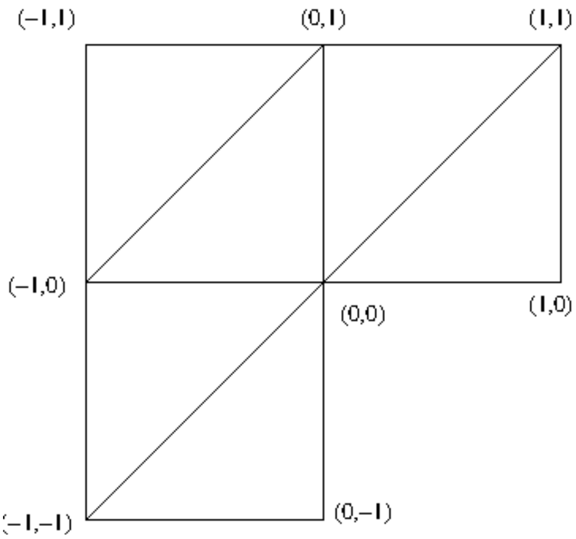
Practical part

6.1. **Approximation of an electric potential.** Solve with ALBERTA the problem

$$
\begin{aligned}
-\Delta u &= 0 && \text{in }]-1,1[^2 \setminus (]0,1[\times]0,-1[),\\
u &= 0 && \text{on } (]-1,1[\times\{1\}) \cup (\{-1\}\times]-1,1[),\\
u &= 1 && \text{on } (]0,1[\times\{0\}) \cup (\{0\}\times]0,-1[),\\
\partial_n u &= 0 && \text{on } (]-1,0[\times\{-1\}) \cup (\{1\}\times]0,1[).
\end{aligned}
$$

by proceeding similarly to Problem 5.1. Moreover:

- Specify the following triangulation in the format of ALBERTA and visualize it:



- Plot the level curve 1/2 in the graphics window of the numerical solution. To this end, do not use the library `gl_tools` but the simple two-dimensional graphics support and insert

  `graph_level(win_val, u_h, 0.5, rgb_black, 0);`

  in the function `graphics` in the file `graphic.c`.

6.2. **Surface distributions.** Let $\Omega \subset \mathbb{R}^d$ be a domain, $\omega \subset \Omega$ a subdomain with Lipschitz boundary and $f \in L^1(\partial\omega)$, where $L^1(\partial\omega)$ indicates the set of all functions that are integrable with respect to the surface measure of $\partial\omega$ (which coincides with the restriction of the $(d-1)$-dimensional Hausdorff-measure to $\partial\omega$). Consider

$$\langle T_f, \varphi \rangle = \int_{\partial\omega} f\varphi$$

for $\varphi \in C_0^\infty(\Omega)$ and verify that it is a distribution. Moreover determine its order and its support.

6.3. **Distributional derivative of piecewise smooth functions.** Let

$$\mathcal{M} : 0 = x_0 < x_1 < \cdots < x_{n-1} < x_n = 1$$

be a mesh of $[0,1]$ and $v : [0,1] \to \mathbb{R}$ a function such that

$$v_i := v_{|]x_{i-1},x_i[} \in C^1[x_{i-1}, x_i].$$

Verify that the distributional derivative of v is

$$v' = \sum_{i=1}^{n} v_i' \chi_i + \sum_{i=1}^{n-1} [v](x_i)\delta_{x_i},$$

where χ_i is the characteristic function of $]x_{i-1}, x_i[$ and we write

$$[v](x_i) := \lim_{\delta \searrow 0} v(x_i + \delta) - v(x_i - \delta)$$

for the jump of v in x_i.

6.4. **Weak Laplacian.** Let Ω be a domain of $\mathbb{R}^d$. Prove that, for any $v \in H^1(\Omega)$, the distributional Laplacian given by

$$\forall \varphi \in C_0^\infty(\Omega) \quad \langle -\Delta v, \varphi \rangle = \langle v, -\Delta\varphi \rangle$$

is (has a unique extension) in $H^{-1}(\Omega)$.

INFORMATION:

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